

# Relaxation and black hole feeding rates in non-spherical galactic nuclei



Eugene Vasiliev  
David Merritt

Rochester Institute of Technology

# Plan of the talk

- Orbits around black holes in non-spherical nuclei
- Difference between spherical, axisymmetric and triaxial nuclear star clusters
- Two-body relaxation in galactic nuclei
- Empty and full loss cone regimes
- Fokker-Planck models and N-body simulations
- Predictions for realistic galaxies; conclusions.

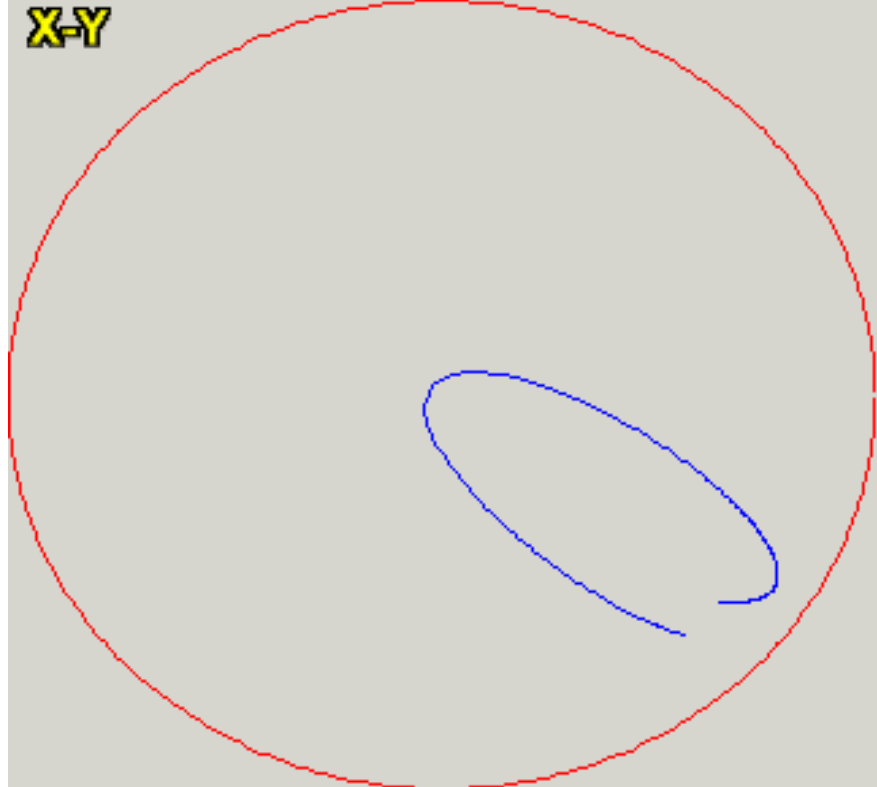
# Nuclear star clusters

- Supermassive black hole  $M_{bh}$
- Stellar cusp (for example, a power law density profile  $\rho \sim r^{-\gamma}$ )
- Total gravitational potential:

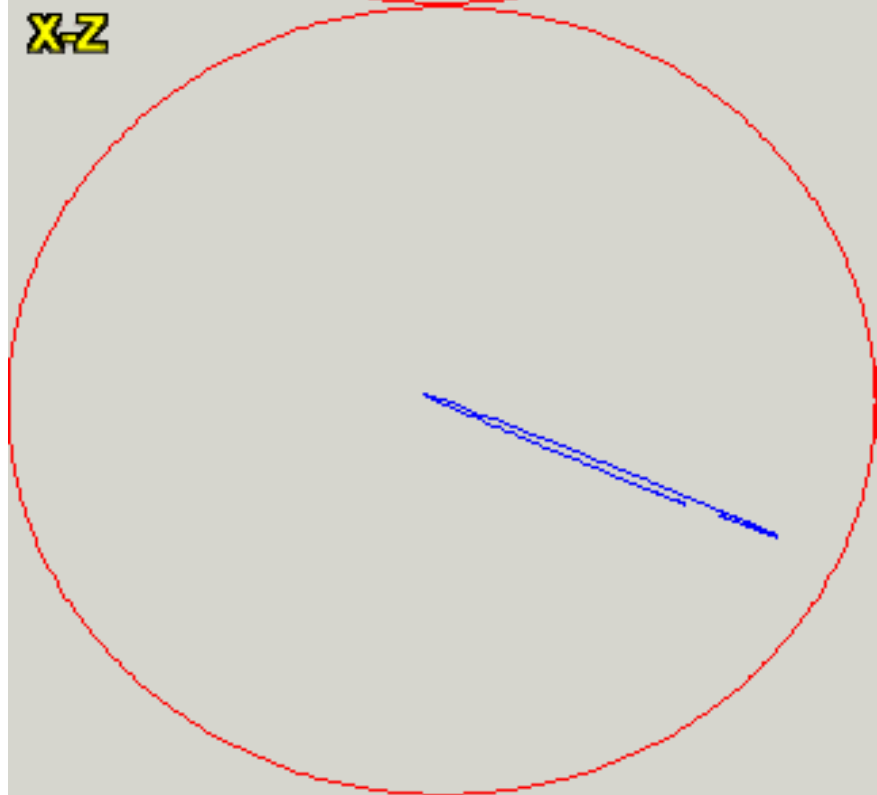
$$\Phi(\vec{r}) = -\frac{GM_{bh}}{r} + \Phi_{\star} \left( \frac{r}{r_0} \right)^{2-\gamma} \left( 1 + \varepsilon \frac{z^2}{r^2} + \eta \frac{y^2}{r^2} \right)$$

- Consider motion inside radius of influence  $r_{infl} \Rightarrow$   
dominant contribution is from SMBH  $\Rightarrow$   
orbits are perturbed Keplerian ellipses  
which precess due to torques from stellar potential  
(motion outside  $r_{infl}$  is discussed towards the end of talk).
- Orbital time  $t_{orb} \ll$  precession time  $t_{prec} \sim r_{infl}/\sigma$

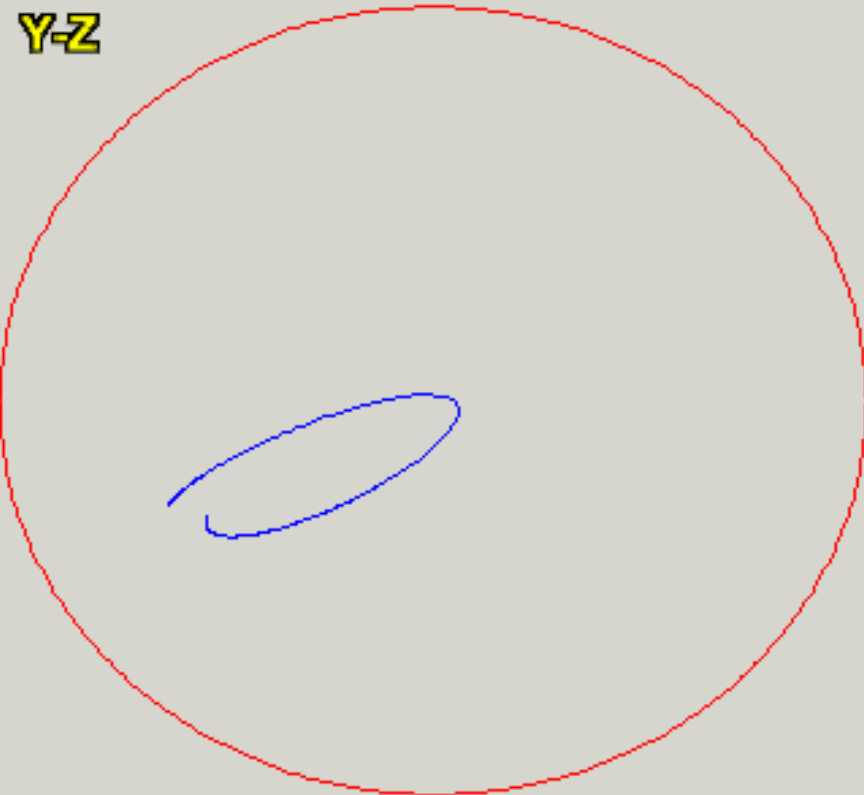
**X-Y**



**X-Z**

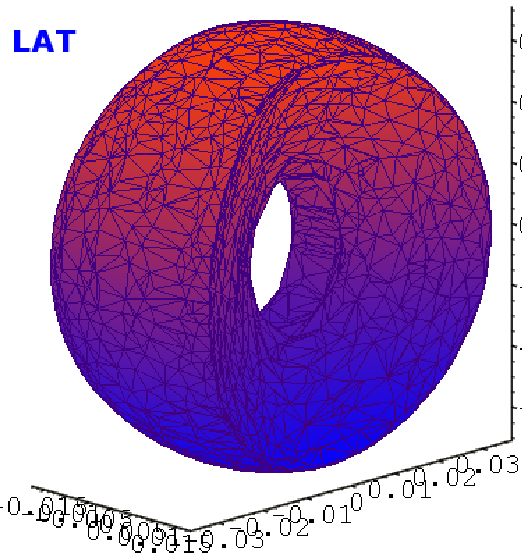


**Y-Z**

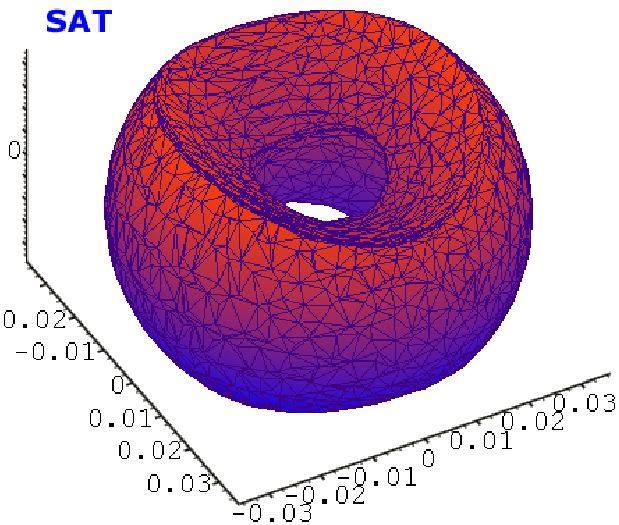


# Types of orbits in non-spherical star cluster around a supermassive black hole

Triaxial cluster:

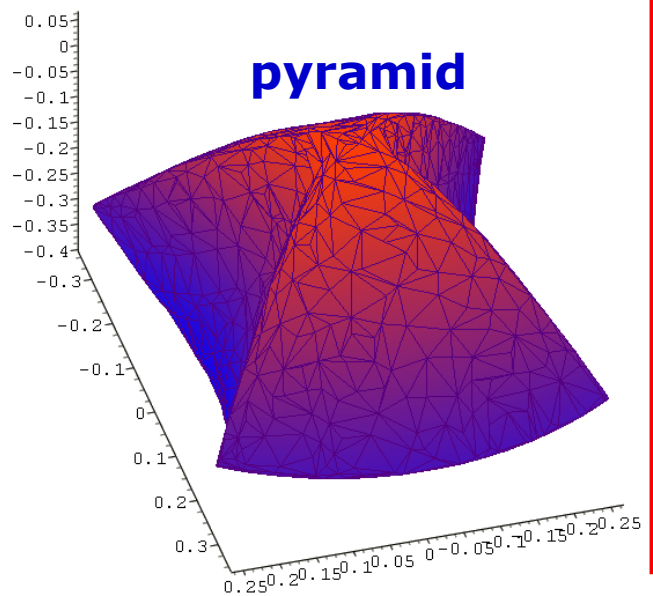


SAT

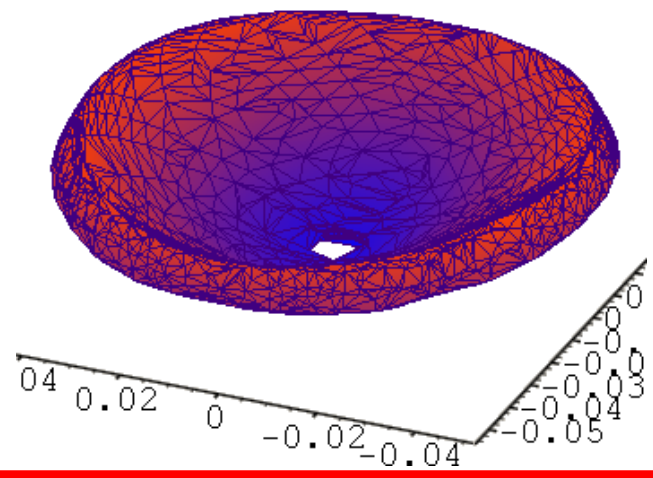


Axisymmetric cluster

pyramid

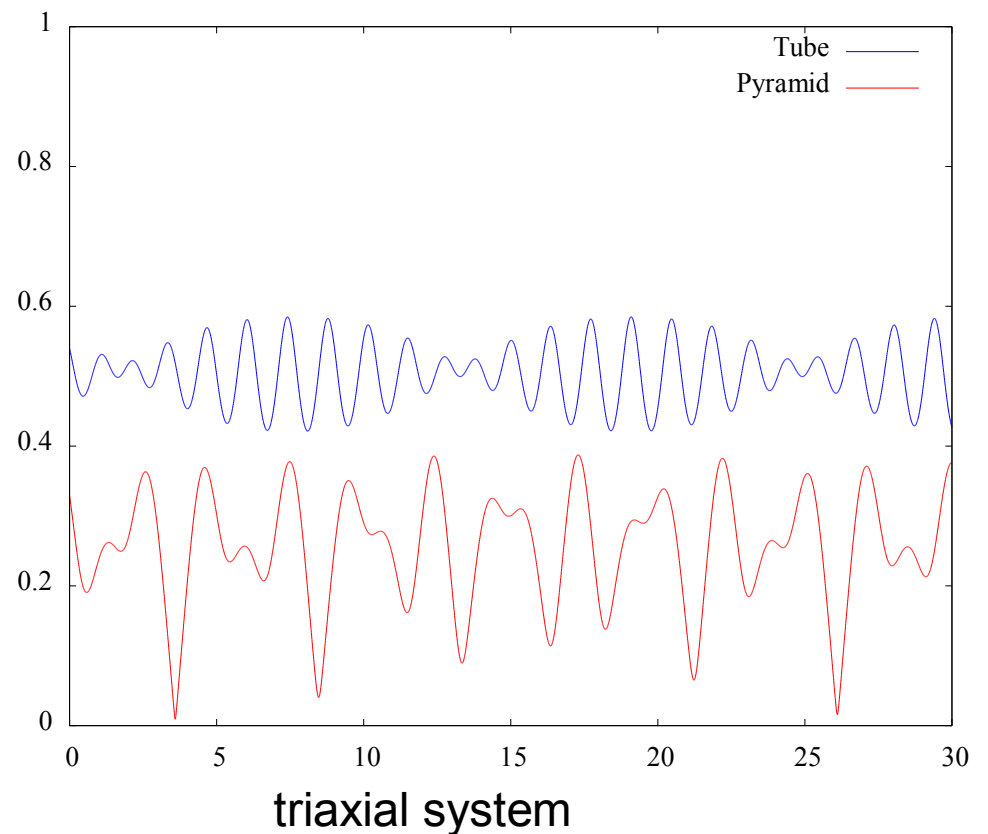
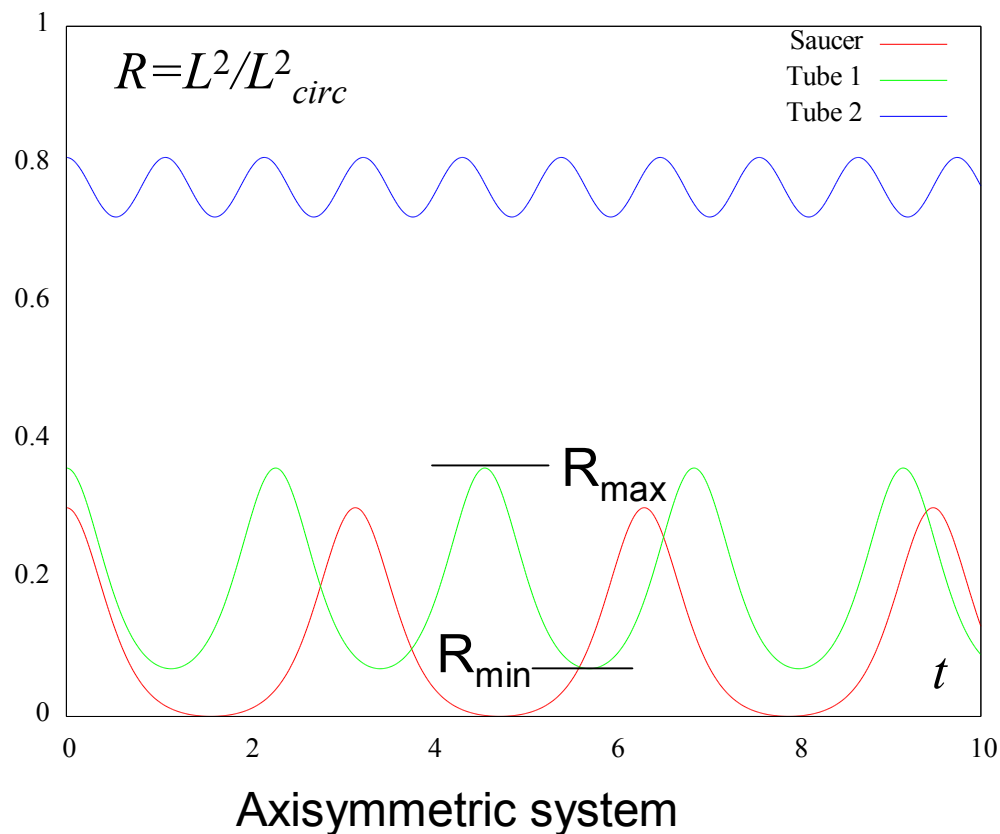


saucer



# Evolution of angular momentum of an orbit in a non-spherical nuclear star cluster

Three integrals of motion: total energy  $E$ , secular hamiltonian  $H$ , and a third integral which is reduced to z-component of angular momentum  $L_z$  in axisymmetric systems. Total angular momentum squared,  $L^2$ , is not conserved but experiences oscillations between  $R_{\min}$  and  $R_{\max}$  with characteristic period  $T_{\text{osc}} \sim T_{\text{prec}}$ , and amplitude  $\sim \varepsilon$ .



# Difference between spherical, axisymmetric and triaxial nuclear star clusters

	Spherical	Axisymmetric	Triaxial
Fraction of stars with $L^2_{\min} < X$	$\propto X$	$\propto \sqrt{X\varepsilon}$	$\propto \varepsilon$
Fraction of time that such a star has $L^2 < X$	1	$\sqrt{X}$	$X$
Survival time of such stars (assuming they are captured immediately after reaching $L^2 < R_{\text{capt}}$ )	$T_{\text{rad}}$ ( $10^{1-5}$ yr)	$T_{\text{osc}}$ ( $10^{5-6}$ yr)	may be longer than $10^{10}$ yr

(for MW nucleus)

but that may not be true  
in the presence of relaxation

# Two-body relaxation in galactic nuclei and the concept of empty/full loss cone

Relaxation time  $T_{rel} = \frac{0.34 \sigma(r)^3}{G^2 \bar{m}_* \rho_*(r) \ln \Lambda}$  – timescale for diffusion in E and L

Loss cone is the region in phase space in which an orbit is captured on the nearest pericenter passage, i.e. at most within 1 radial period, having  $L^2/L_{circ}^2 = R < R_{lc}$ .

The question is how fast the changes in L occur compared to radial period:

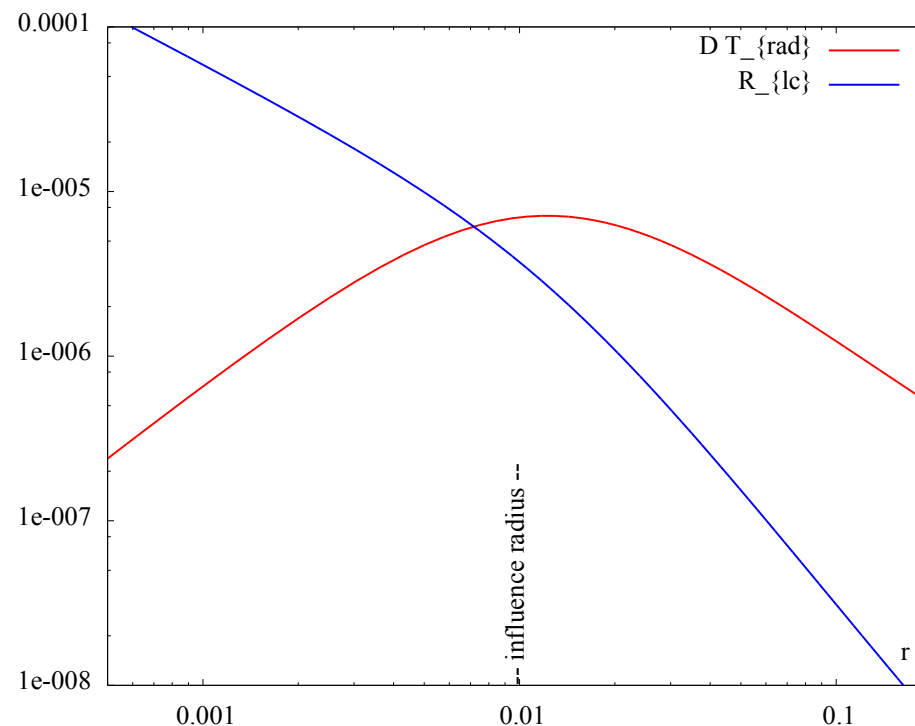
$$q = \Delta R^2 / R_{lc}^2,$$

$q \ll 1$  – empty loss cone regime:

stars are captured as soon as they enter LC;  
population of stars with  $L^2 < R_{lc}$  is negligible

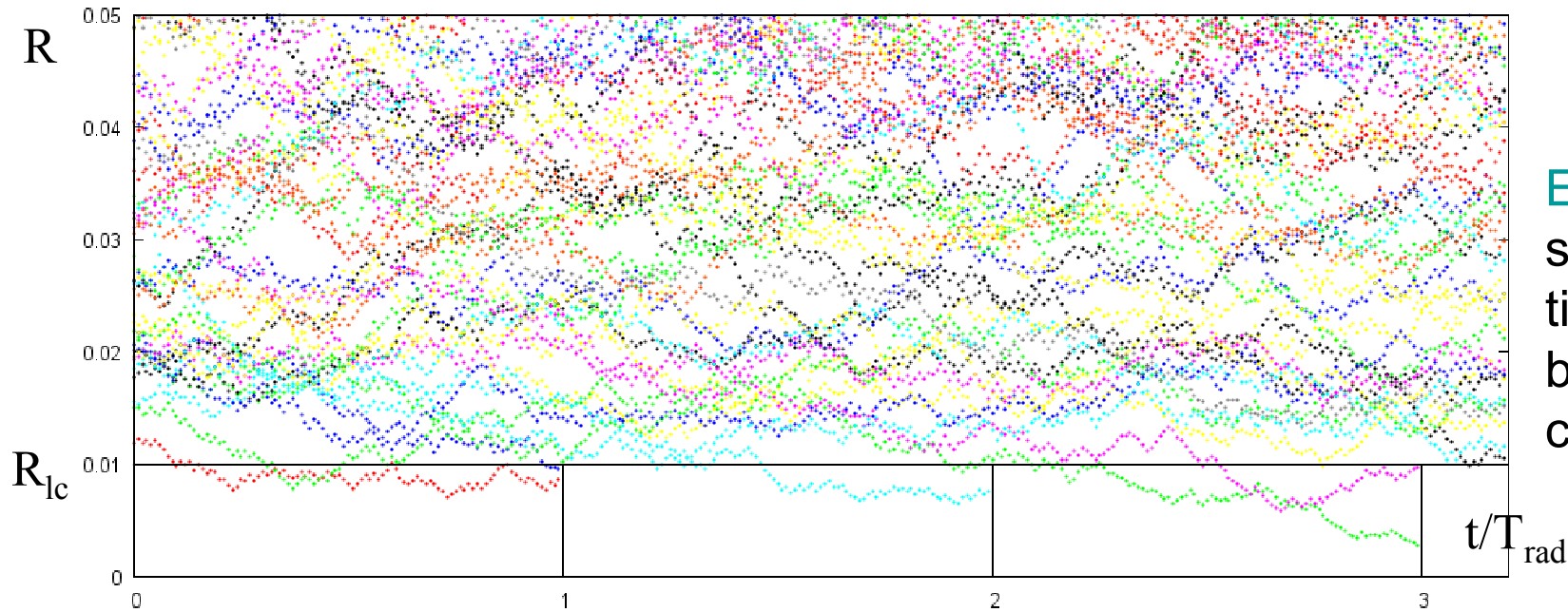
$q \gg 1$  – full loss cone:

stars may move in and out of LC many times  
before being captured at the end of  $T_{rad}$ ,  
d.f. of stars in LC is the same as elsewhere



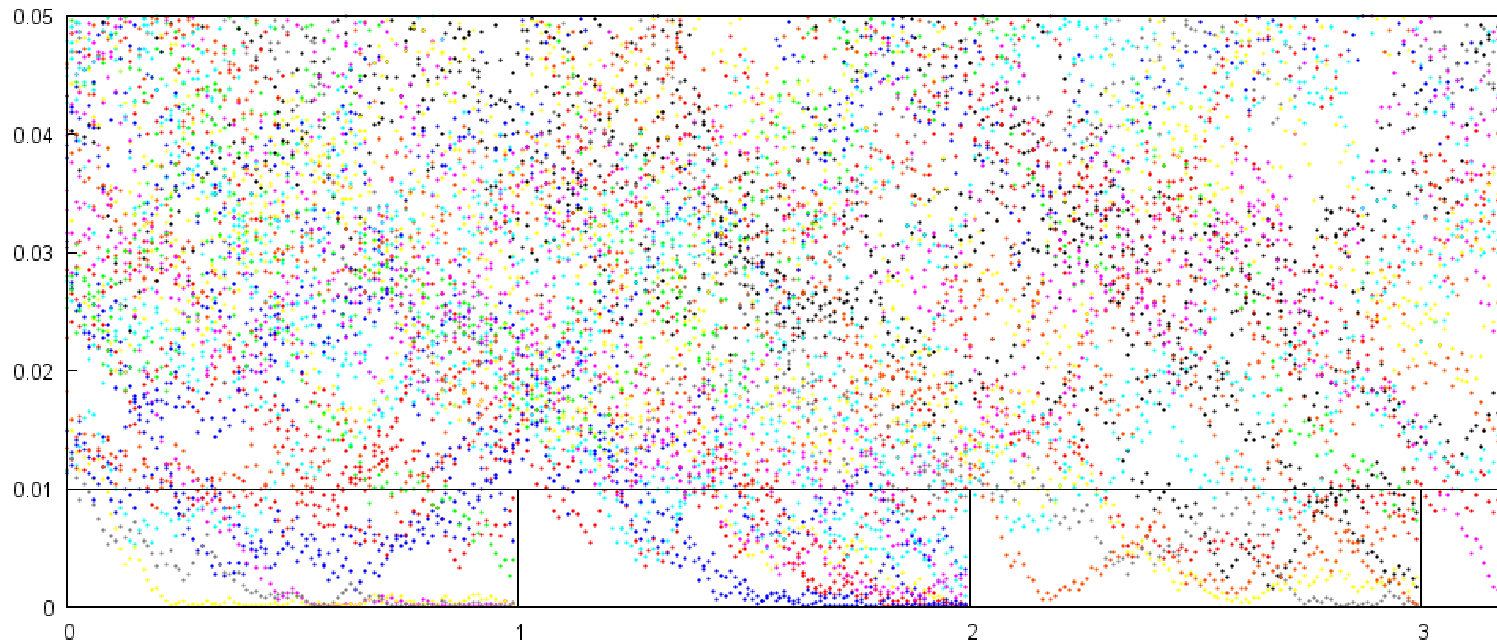


# The concept of empty/full loss cone

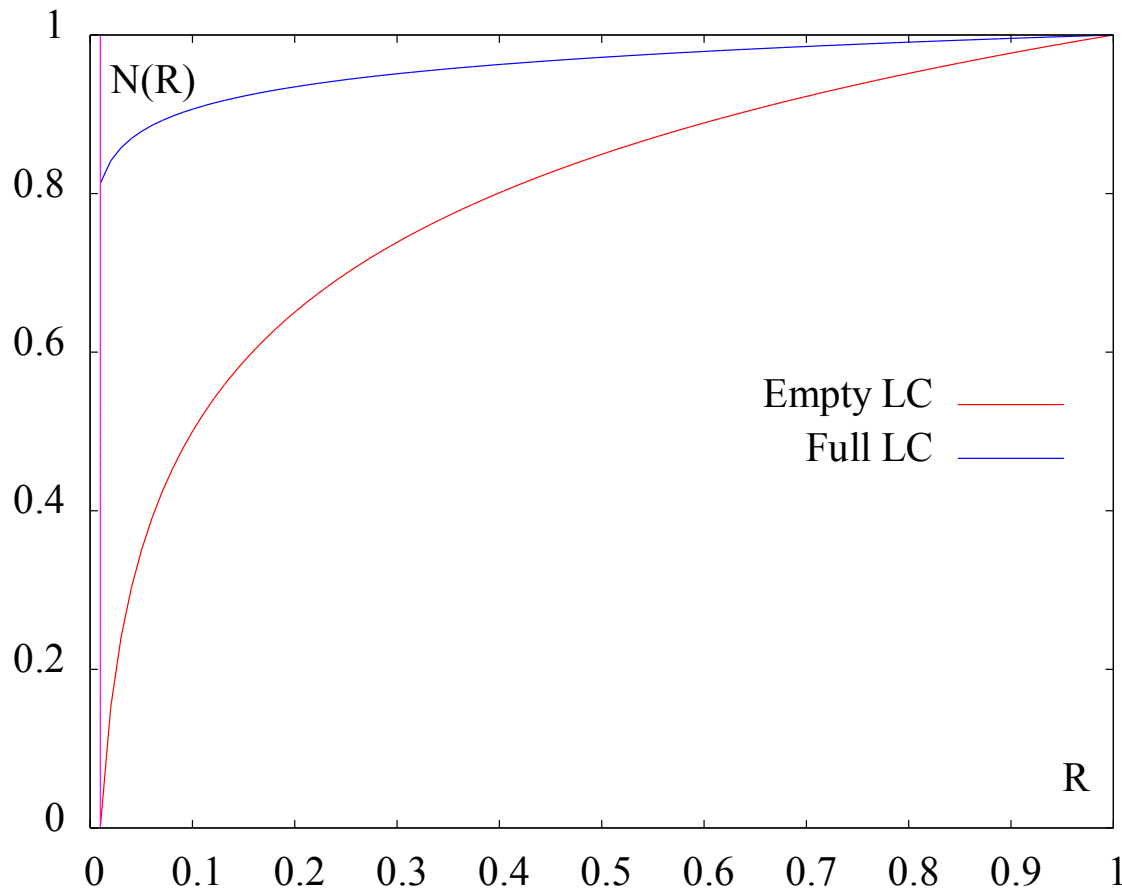


**Empty LC:**  
stars barely have time to enter LC before they get captured after  $T_{\text{rad}}$

**Full LC:**  
stars may enter and exit LC many times during one  $T_{\text{rad}}$



# The concept of empty/full loss cone



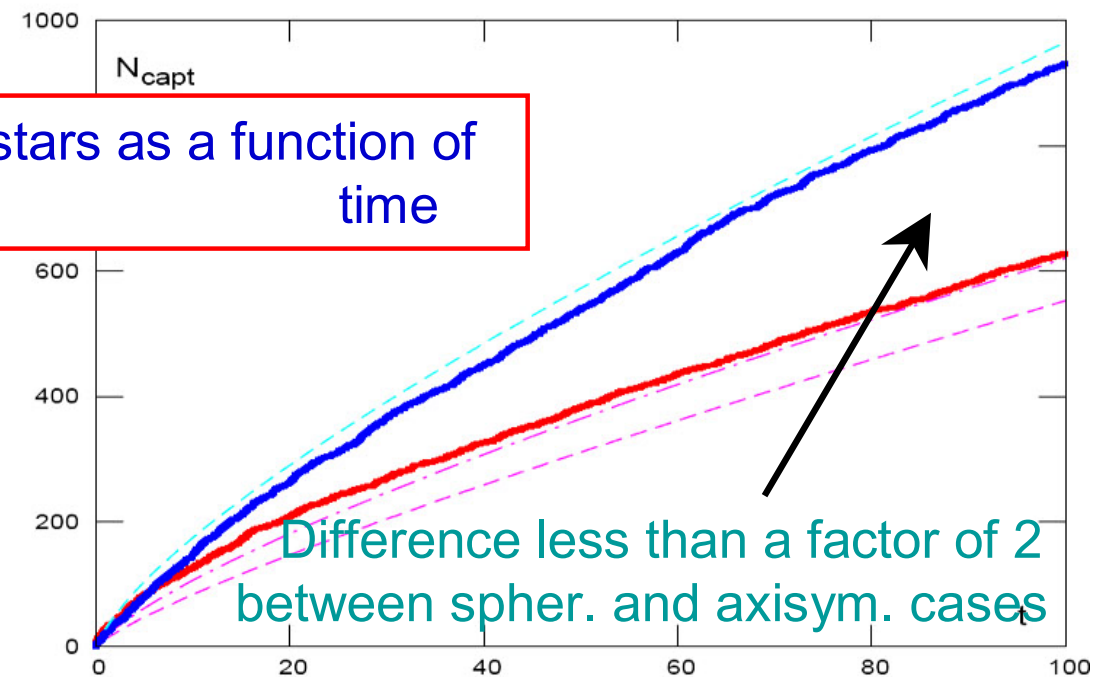
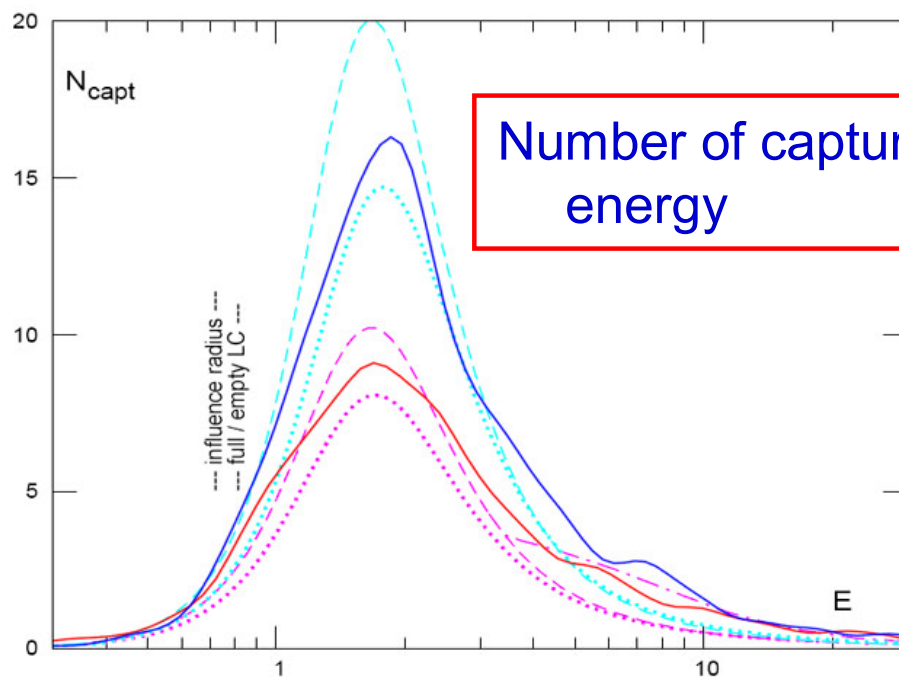
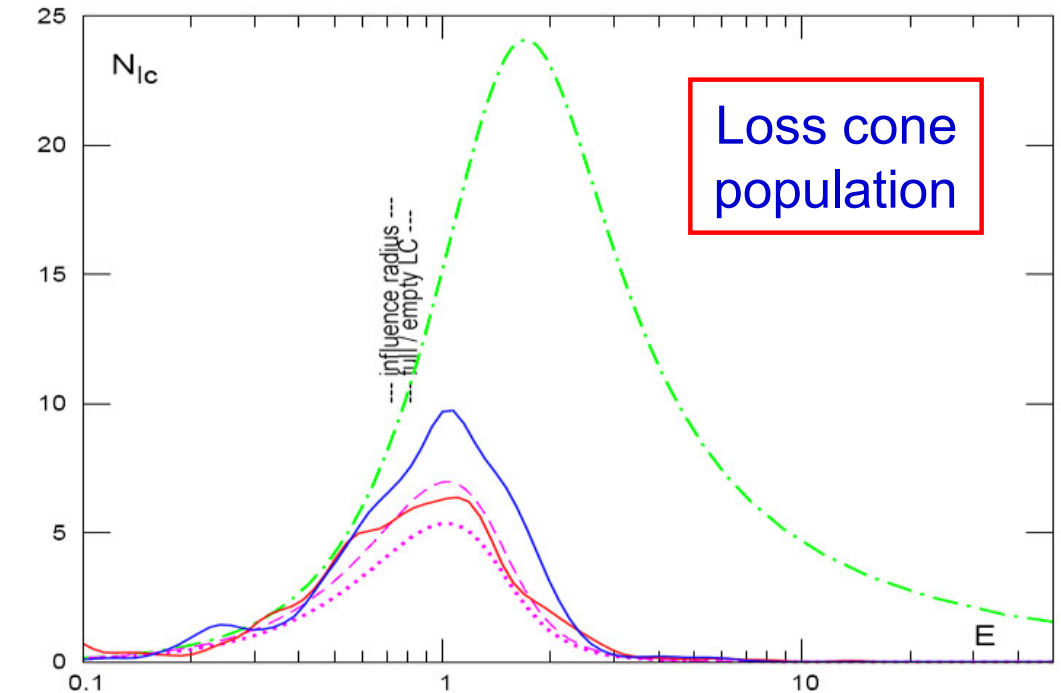
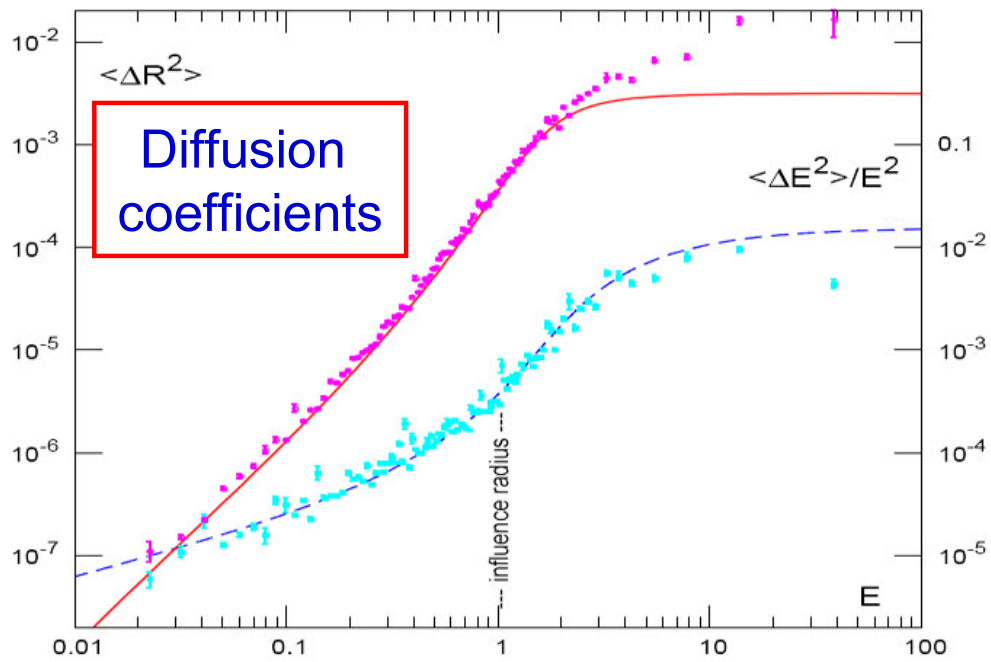
- In the empty LC regime,  $N(R_{lc}) \sim 0$ ,  $N(R) \sim \log R$ , capture rate is limited by diffusion (gradient of  $N(R)$ ):  $F \sim T_{rel}^{-1} / (\log(1/R_{lc}) - 1)$  for standard 2-body relaxation
- In the full LC regime,  $N(R_{lc}) \sim N(R) \sim 1$ , capture rate is  $F \sim R_{lc} / T_{rad}$

does not depend on diffusion coefficient or even on the mechanism of LC refill as long as it is efficient enough to keep it full!

# Loss cone draining vs. relaxation

- Regular precession may shuffle stars in angular momentum more efficiently than 2-body relaxation
- The capture rate cannot exceed  $F_{\text{full LC}}$ , but can be larger than in the spherical case if it was in the empty loss cone regime
- After all orbits with  $L_{\text{min}}^2 < R_{\text{capt}}$  have been drained, the influx of stars from higher  $L$  is still limited by diffusion (relaxation in angular momentum)
- For triaxial nuclei, the draining time of pyramid orbits may be  $>10^{10}$ yr. For axisymmetric systems, adequate description of relaxation is needed (in terms of Fokker-Planck equation in terms of the variables which are integrals of motion in the absence of relaxation).
- Comparison with N-body simulations to determine applicability of F-P description; extrapolation of F-P results into the range of parameters inaccessible for direct N-body.

# Comparison of Fokker-Planck models with N-body simulations



Number of captured stars as a function of energy

Difference less than a factor of 2 between spher. and axisym. cases

# Conclusions

- In non-spherical nuclear star clusters the star angular momentum  $L$  is changed not only due to 2-body relaxation, but also due to regular precession
- This facilitates the capture of stars at low  $L$ :  
the “expanded” loss region is where  $L_{\min}^2 < L_{\text{capt}}^2$ , not just  $L^2 < L_{\text{capt}}^2$
- Draining time of this region is  $\sim T_{\text{prec}} \sim 10^{5-6}$  yr in axisymmetric case and much longer, comparable to Hubble time, in triaxial case
- Compared to the spherical case, the difference in total capture rate for axisymmetric case is relatively small ( $\sim$ factor of 2) and is important only in the transition regime between empty and full loss cone
- For giant elliptical galaxies, which are deeply in the empty loss cone regime for a spherical case, the enhancement may be more dramatic