Beyond collisionless Boltzmann equation: when relaxation is important

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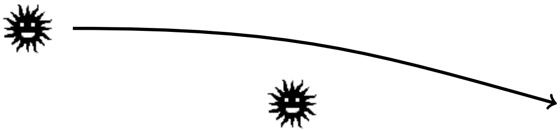
Two-body relaxation

- Classical Chandrasekhar/Spitzer two-body relaxation theory:



- Relaxation time in hot stellar systems: $T_{\text{rel}} \propto \frac{N}{\ln N} T_{\text{dyn}}$
- For galaxies or galactic nuclei, $T_{\text{rel}} \gg T_{\text{Hubble}} \sim 10^{10}$ yr
- Galaxies have $N \sim 10^{8\ldots 11}$, but N-body simulations – only $N \sim 10^{5\ldots 7}$
- The enhancement of two-body relaxation in N-body simulations could severely distort physical processes even if $T_{\text{rel,N-body}} \gtrsim T_{\text{Hubble}}$

Example 1: evolution of supermassive black hole binaries

galaxy merger

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slingshot ejection

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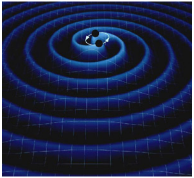
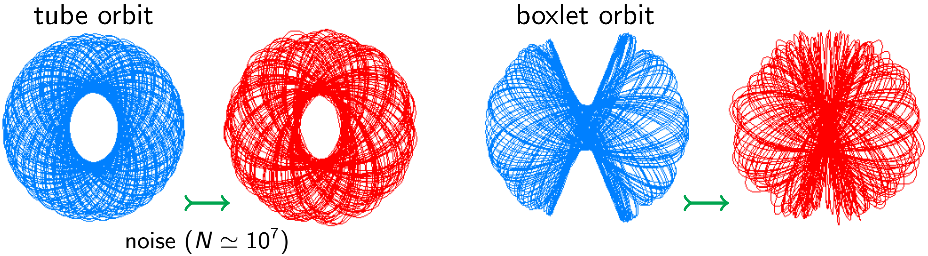


BH coalescence

- Binary BH ejects stars in 3-body scattering events and shrinks
- Interaction rate depends on the population of stars in the loss cone
- Loss cone is refilled both by collisional relaxation and collisionless processes (non-spherical torques)
- Collisional repopulation time $\propto (a_{\rm bin}/a_\star)\, T_{\rm rel} \ll T_{\rm rel}$
- This timescale is still $\gg T_{\rm Hubble}$ in real galaxies, but no longer so in N-body simulations!

Example 2: chaotic diffusion in triaxial potentials



- Triaxial galactic potentials support a large variety of orbit types
- Many of them belong to minor resonant families or are weakly chaotic
- Noise greatly increases the diffusion of chaotic orbits in phase space [Kandrup+ 2000s]
- This could lead to a rapid loss of triaxiality

Smooth-field method

Spherical-harmonic expansion for the global stellar potential

(cf. Aarseth 1967, Hernquist&Ostriker 1992) :

$$\Phi(r,\theta,\phi) = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^{l} \Phi_{l,m}(r) Y_l^m(\theta,\phi);$$

$$\Phi_{l,m}(r) = -\frac{4\pi G}{2l+1} \left[r^{-l-1} \sum_{r_i<r} m_i Y_l^m(\theta_i,\phi_i) r_i^l + r^l \sum_{r_i>r} m_i Y_l^m(\theta_i,\phi_i) r_i^{-1-l} \right]$$

- Particles do not interact directly, but only through the smooth mean field
- Scaling: $\mathcal{O}(N)$
- May enforce a particular kind of symmetry
  (e.g. axisymmetry – keep only terms with even l and $m = 0$)
- Suitable for isolated, not too flat systems with a well-defined center

Suppression of discreteness noise

- **Spatial smoothing:**

  keep only a few low-order terms in spherical-harmonic expansion,

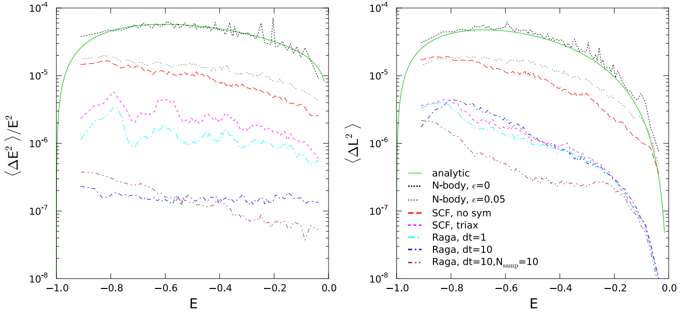
  use optimally-smoothed spline in radius

- **Temporal smoothing:**

  update the mean-field potential not too frequently ($T_{\rm dyn} \lesssim T_{\rm upd} \ll T_{\rm rel}$)

- **Oversampling:**

  take $N_{\rm samp} \gg 1$ points from each particle's trajectory during each interval $T_{\rm upd}$



Result: $\sim 100\times$ suppression of fluctuations

Addition of two-body relaxation

- Goal:
  simulate the effect of collisional relaxation in a probabilistic way
- Method:
  add random perturbations to particle velocities after each timestep [Spitzer 1970s, Rosenbluth+ 1957]:

$$\Delta v_{\parallel} = \langle \Delta v_{\parallel} \rangle \Delta t + \zeta_1 \sqrt{\langle \Delta v_{\parallel}^2 \rangle \Delta t}, \quad \Delta v_{\perp} = \zeta_2 \sqrt{\langle \Delta v_{\perp}^2 \rangle \Delta t}, \qquad \zeta_1, \zeta_2 \sim \mathcal{N}(0,1)$$

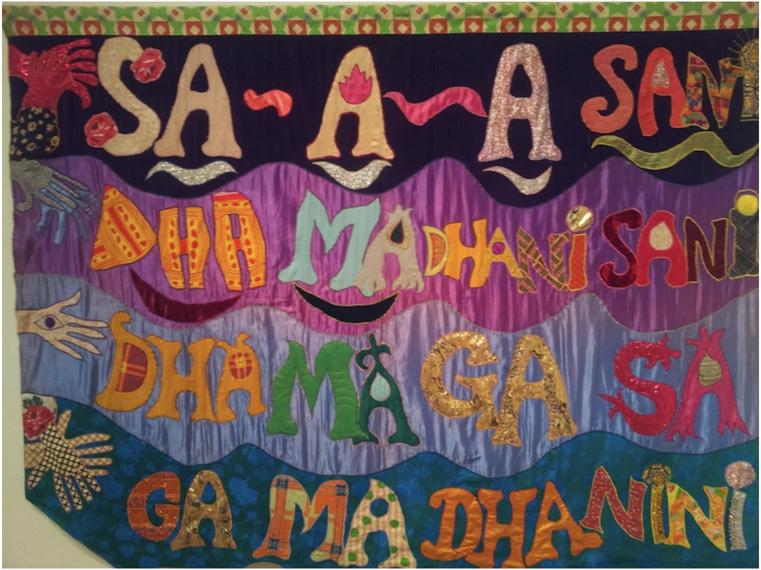
$$v \langle \Delta v_{\parallel} \rangle = -\left(1 + \tfrac{m}{m_\star}\right) I_{1/2}, \quad \langle \Delta v_{\parallel}^2 \rangle = \tfrac{2}{3} \left(I_0 + I_{3/2} \right), \quad \langle \Delta v_{\perp}^2 \rangle = \tfrac{2}{3} \left(2 I_0 + 3 I_{1/2} - I_{3/2} \right)$$

$$I_0 \equiv \Gamma \int_E^0 dE' \, f(E'), \quad I_{n/2} \equiv \Gamma \int_{\Phi(r)}^E dE' \, f(E') \left(\frac{E' - \Phi(r)}{E - \Phi(r)} \right)^{n/2}$$

$$\Gamma \equiv 16 \pi^2 G^2 M_{\mathrm{tot}} \times \boxed{(N_\star^{-1} \ln \Lambda)}$$

- Motivation:
  adjustable amount of relaxation, decoupled from the number of particles;
  may compare with N-body simulations by setting $N_\star = N$,
  or consider [nearly-] collisionless limit by turning off perturbations

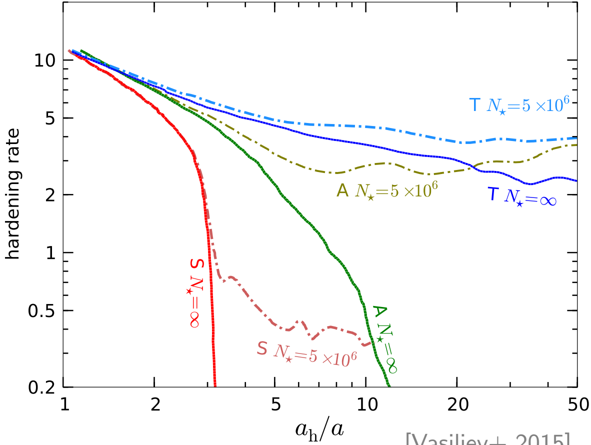
raga – relaxation in any geometry



[Moki Cherry, "Raga", 1970s]

Results: binary black hole evolution

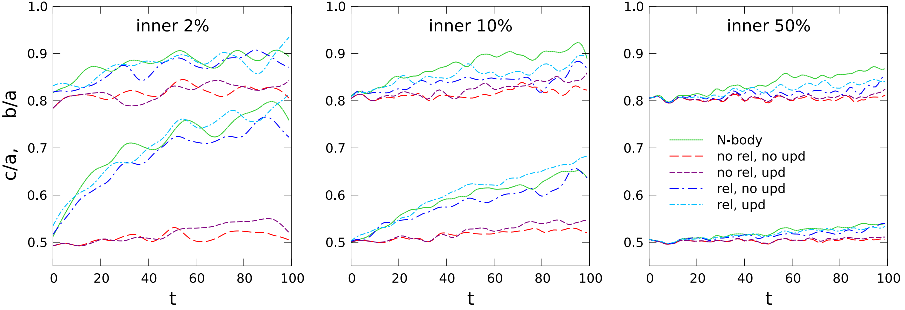
- In the collisionless case, the binary continues to shrink only if the galaxy is triaxial, but not in axisymmetric or spherical geometries
- Addition of relaxation (at the level of state-of-the-art N-body simulations) qualitatively changes the evolution in the latter two cases



[Vasiliev+ 2015]

Results: chaotic diffusion in triaxial galaxies

- Without relaxation, the shape of the system remains stable
- Addition of relaxation accelerates diffusion of chaotic orbits and leads to a gradual loss of triaxiality
- Good agreement between Monte Carlo and N-body codes for the same amount of relaxation



[Vasiliev 2015; Hamilton & Vasiliev, in prep.]

Summary

- Two-body relaxation can play a substantial role even in stellar systems with very large N
- No N-body code is truly collisionless, but one can strongly suppress noise in slowly evolving systems
- Software is available at `https://github.com/GalacticDynamics-Oxford/Agama`

Merci pour votre compréhension!