Orbital analysis of N-body simulations and galactic models:

what can we learn about stability and evolution of these models?

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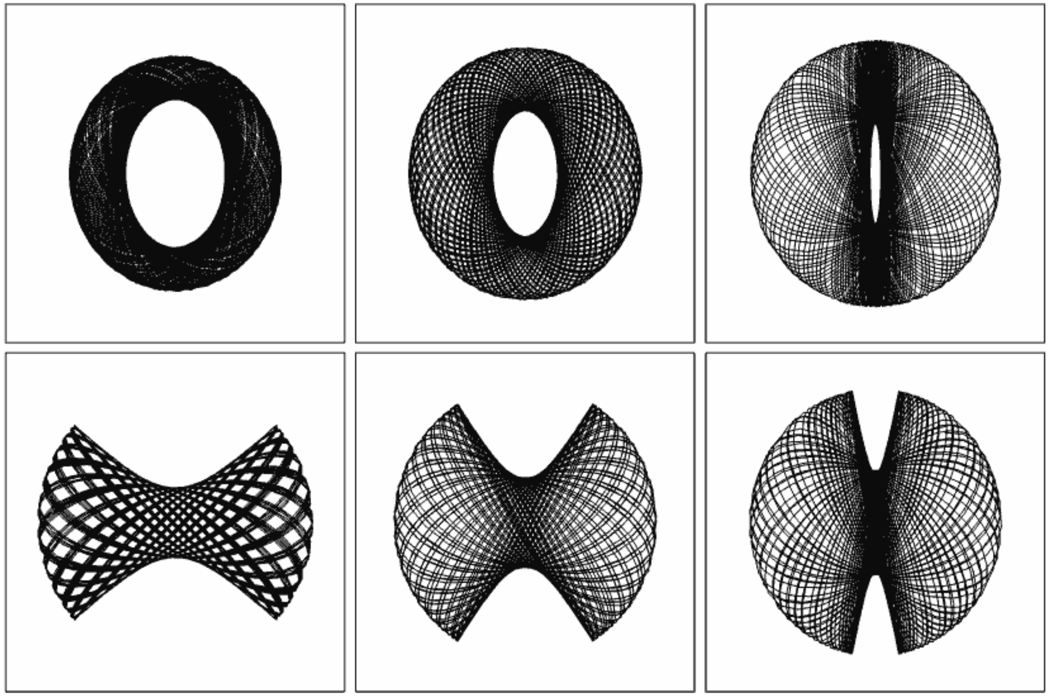
Plan of talk

- Basic facts about orbits of stars in [triaxial, nonrotating] galaxies
- Regular and chaotic orbits
- Resonant orbits and their importance for global dynamics
- Orbit analysis in *N*-body models
- Applications
  - – stability of triaxial cuspy galaxies
  - – centrophylic orbits and supermassive black holes
  - – adiabatic compression and shape evolution in galaxy formation

Types of orbits in 2d integrable potential

Archetypical planar potential:

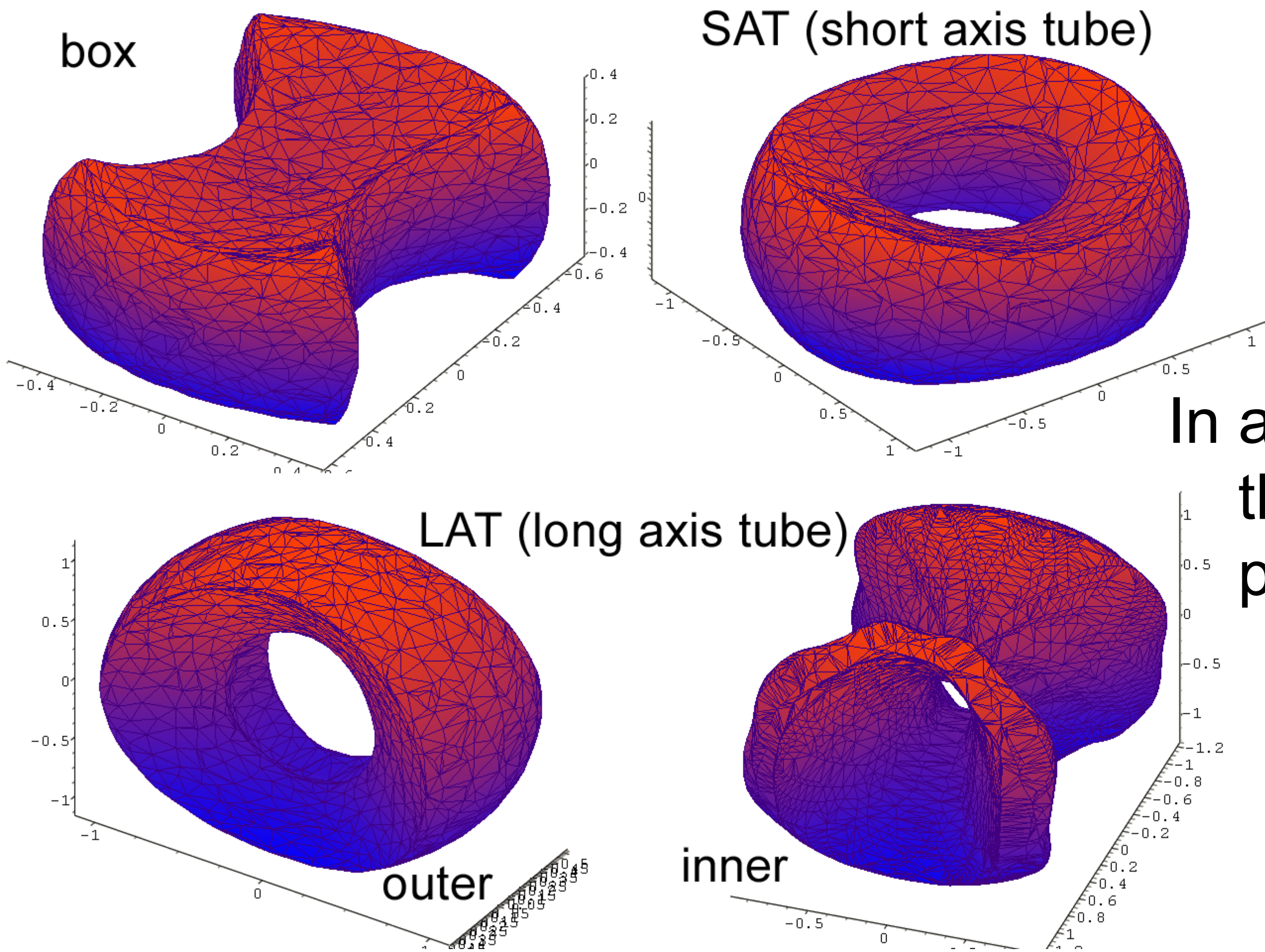
$$\Phi(x,y) = \frac{1}{2}v_0^2 \ln(1+m^2)\ , \quad m^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}\ , \quad a \geq b$$



Loop orbits:
circulation about origin
(or epicyclic motion around
a closed nearly-circular orbit)

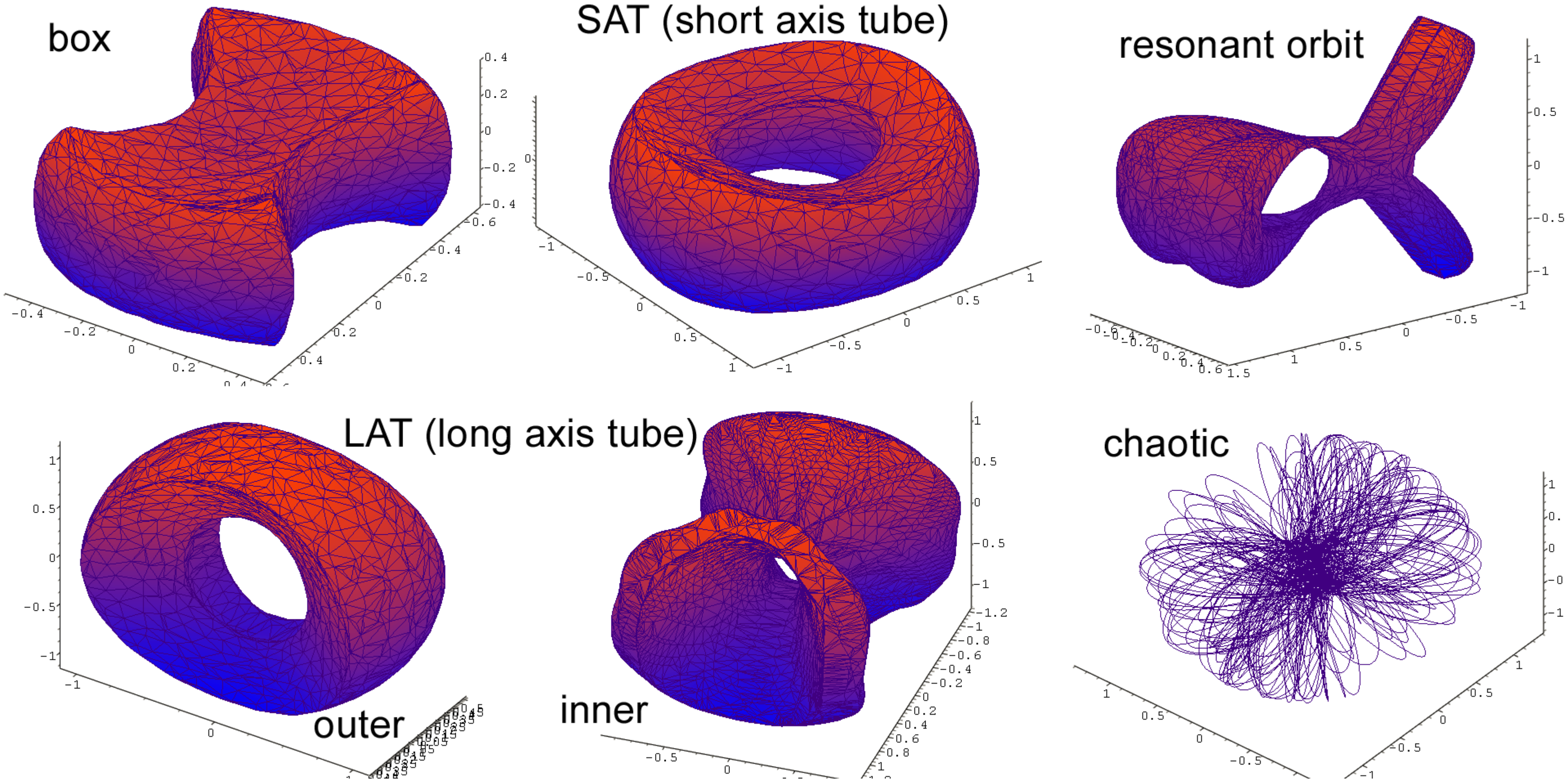
Box orbits:
motion in a distorted rectangle;
no definite sense of rotation

Types of orbits in 3d integrable potential



In an integrable potential these are the only possible orbit types

Types of orbits in 3d non-integrable potential



Regular and chaotic orbits

In a system with N degrees of freedom

a regular orbit has N integrals of motion,

a chaotic one has less than N.

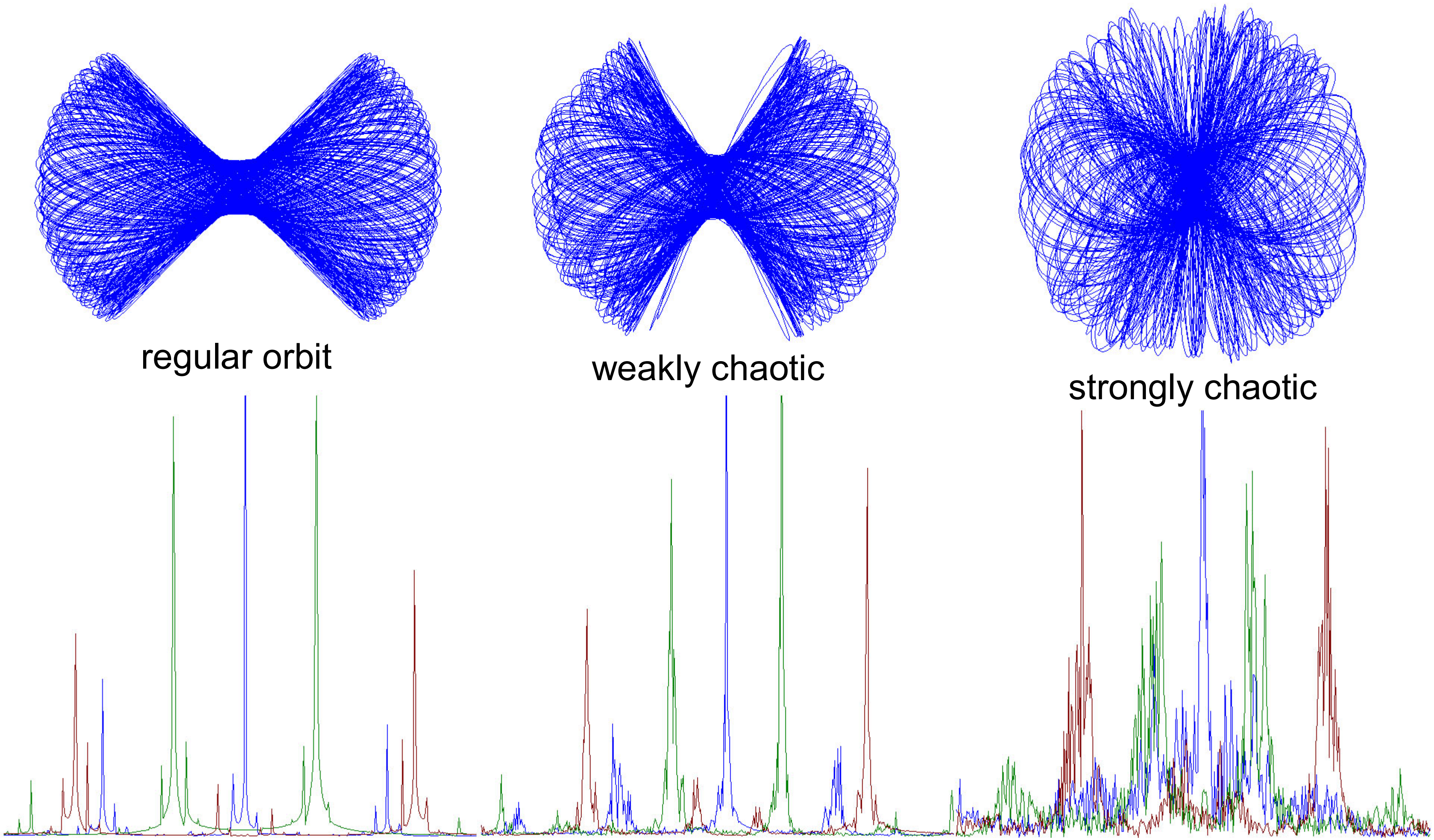
But the integrals are rarely known in explicit form!

(for time-independent potential the total energy E is the only classical integral)

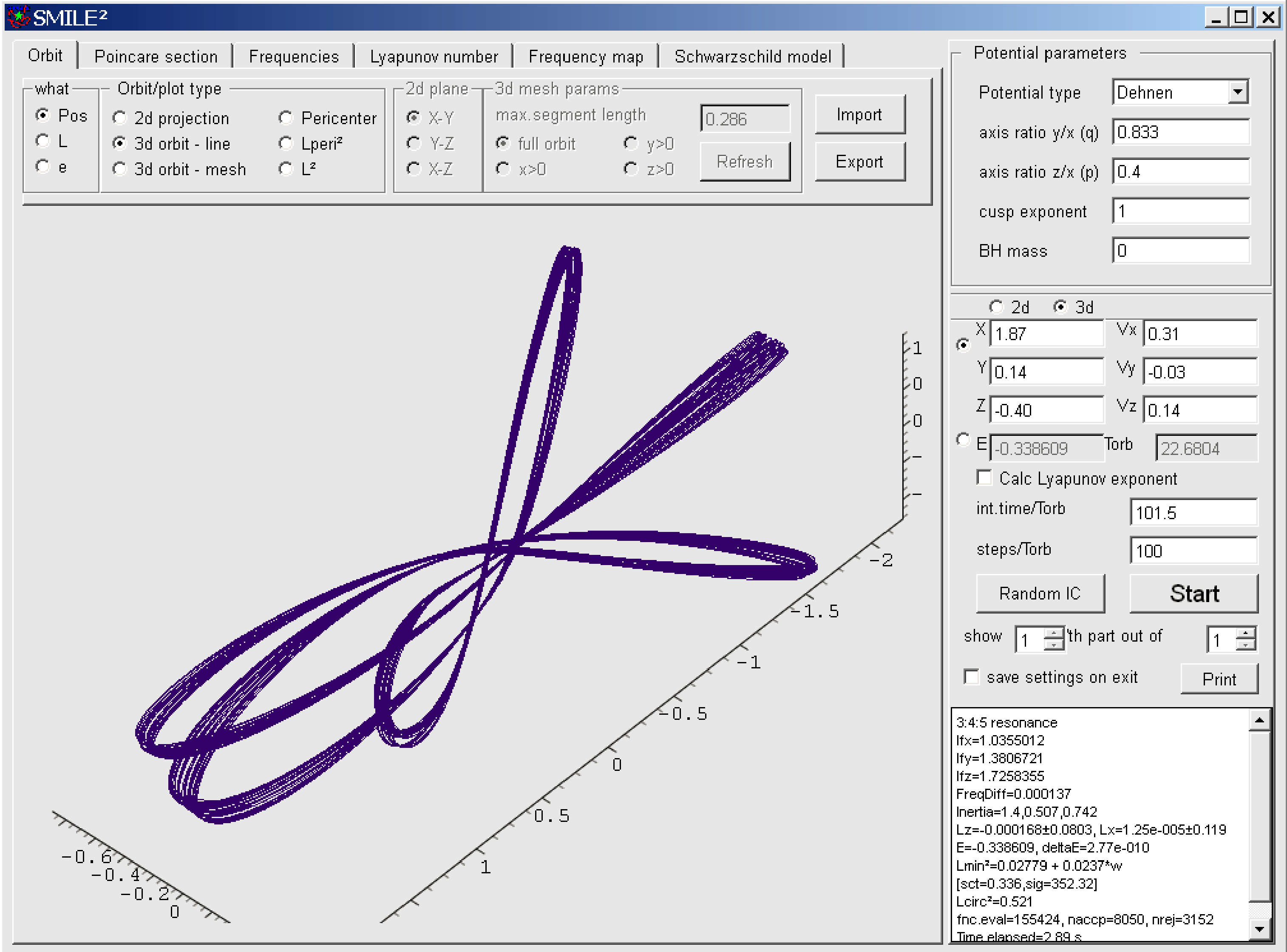
Methods for analysis of orbit stochasticity:

- Lyapunov exponent
- "Diffusion rate" of fundamental frequencies of motion
- Smallest alignment index (SALI), etc…

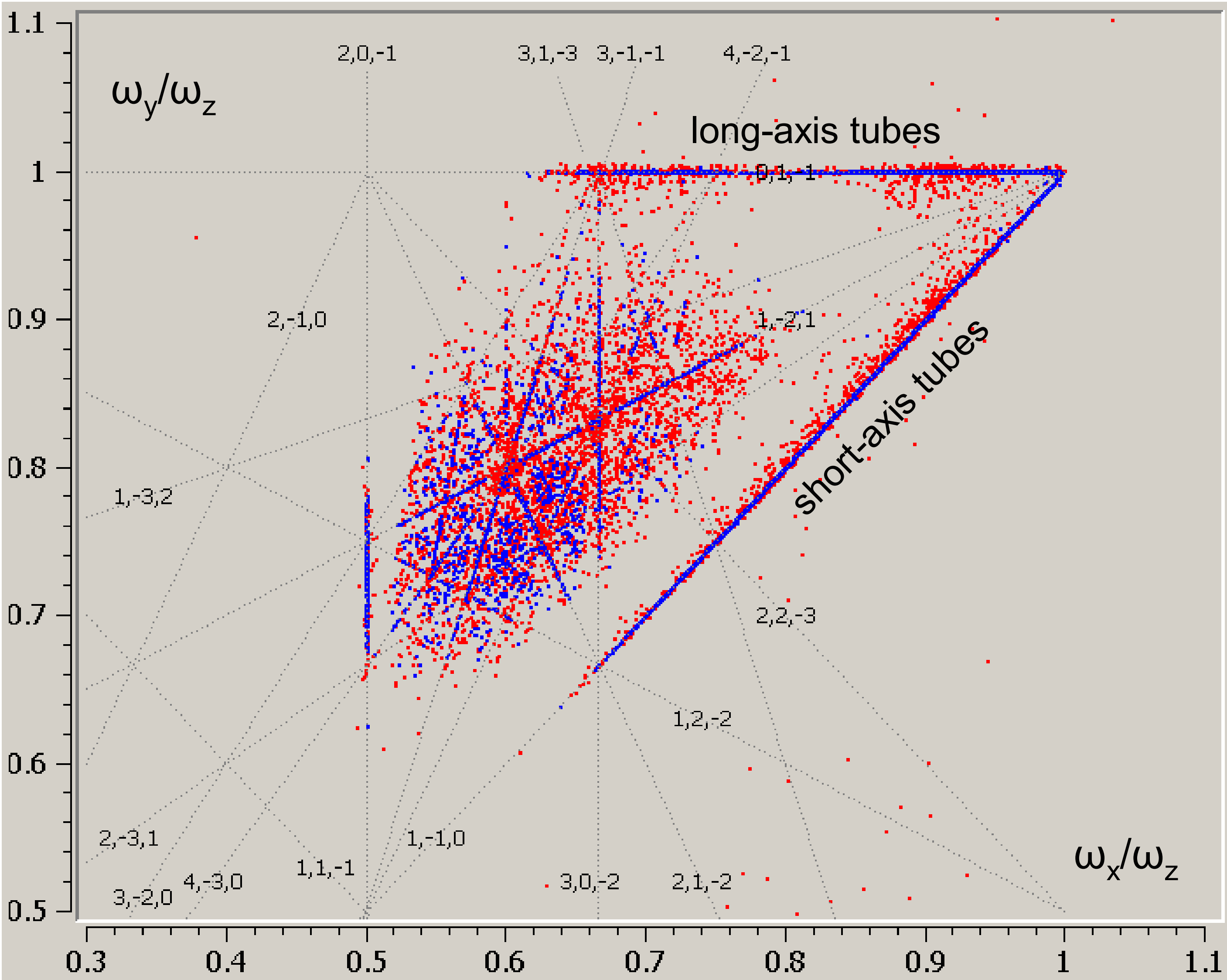
No well-defined transition to chaos



Resonant orbits

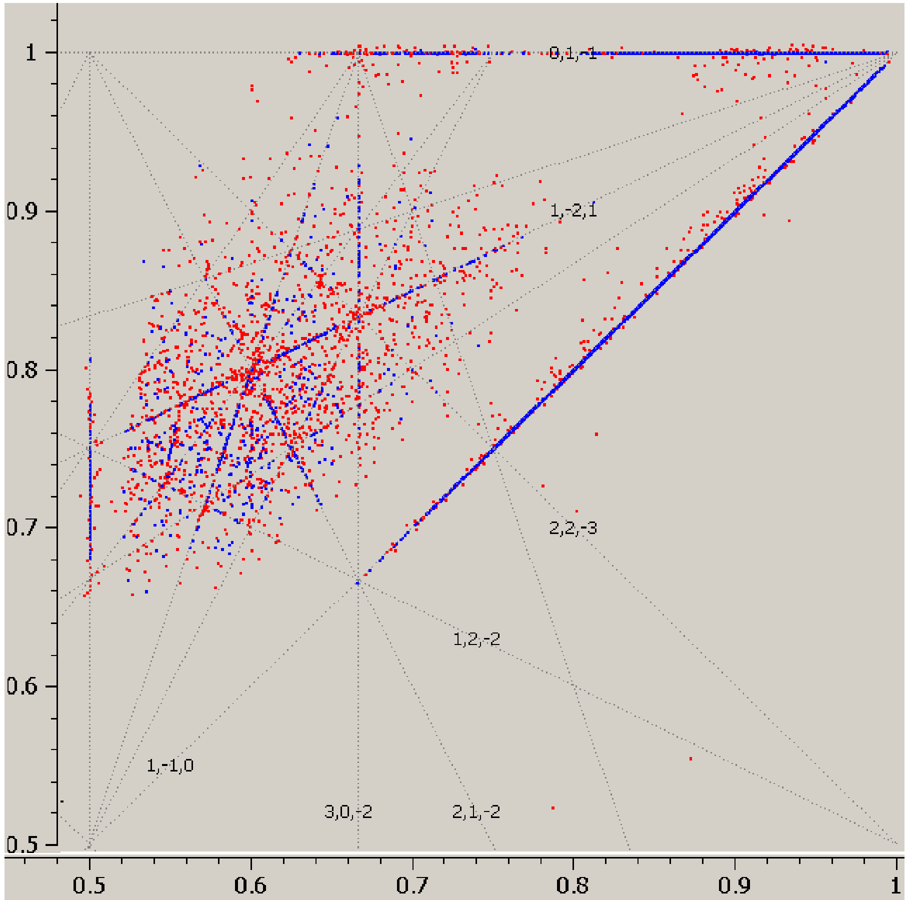


Frequency maps

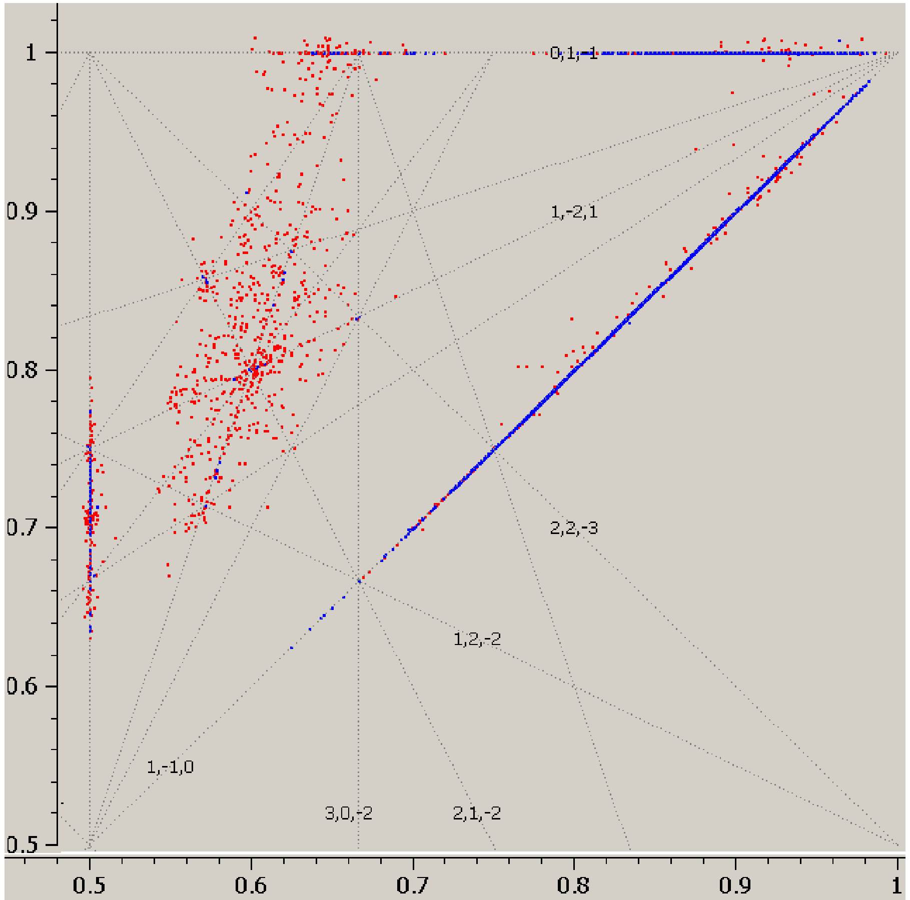


Frequency maps as tools for studying global dynamics

γ=1 Dehnen model:
many resonant orbits that create the structure of the phase space

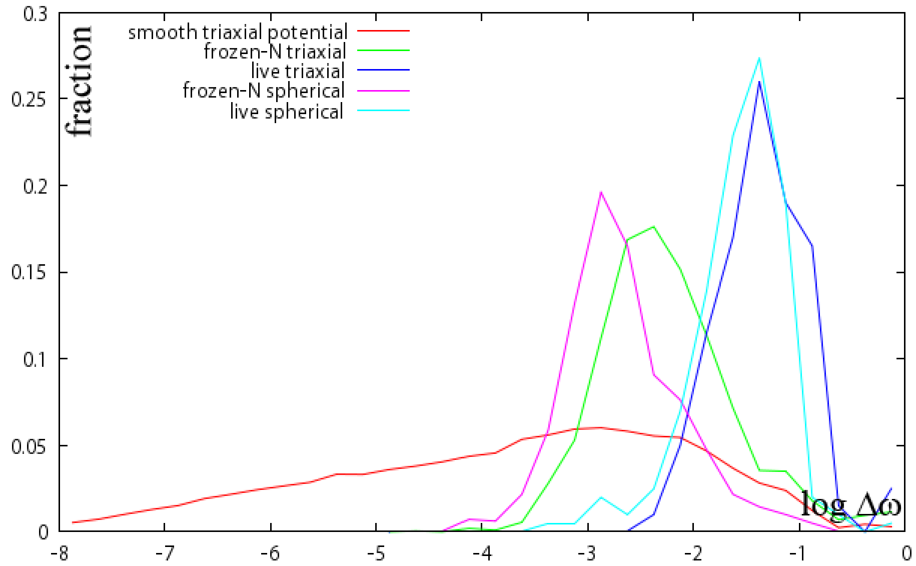


γ=2 Dehnen model:
almost all non-tube orbits are chaotic;
no global barriers to chaotic diffusion



Orbit analysis in N-body simulations

- Objects (galaxies, dark matter halos, etc) in simulations (and likely in reality as well) are not stationary, but evolve in time
- Even in a quasi-stationary "live" model, two-body relaxation means that orbit energy is not conserved
- Even if we study motion of a test particle in a "frozen" N-body potential, discreteness effects induce irregularities in motion (e.g. Lyapunov exponents are always positive; frequency diffusion rates are much higher than in a smooth potential; resonances are smeared out)



Representation of N-body model with a smooth potential

- **Option 1**: approximate by a known analytical model (with best-fit parameters)
- **Alternative:** use a generic form of approximation with tunable coefficients

Spherical-harmonic expansion of density and potential:

$$\rho(r,\theta,\phi) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} A_{lm}(r)\, P_l^m(\cos\theta)\, \cos m\phi$$

- **Option 2:**

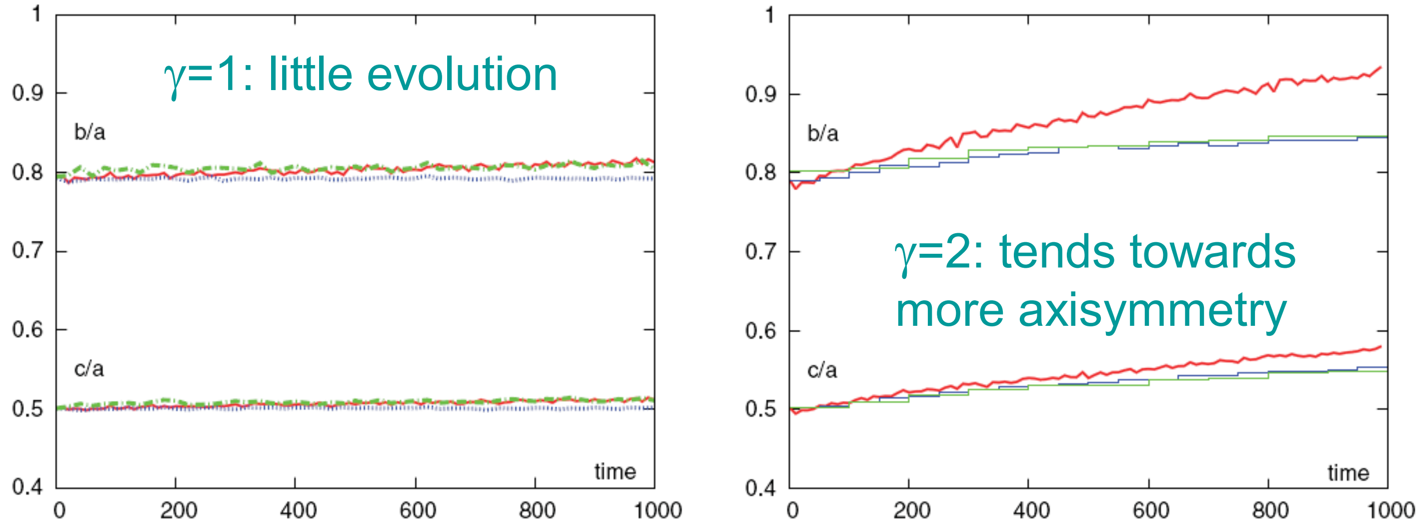
Bi-orthogonal basis-set: $A_{lm}(r) = \sum_{n=0}^{n_{max}} \boldsymbol{A}_{nlm}\, B_{nl}(r)$

- **Option 3:**

Spline expansion in radius: $A_{lm}(r)$ is a smooth interpolating function

Applications: chaotic diffusion and secular evolution of triaxial cuspy galactic models

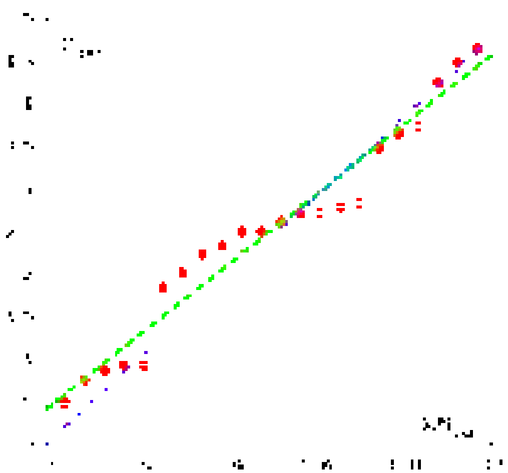
- Construct models of cuspy triaxial elliptical galaxies with Schwarzschild's orbit superposition method
- Explore orbital structure of γ=1 and γ=2 models; the first has rich network of resonant orbits, the second has none
- Study evolution of shape in fixed-potential integrations and in a "live" N-body system

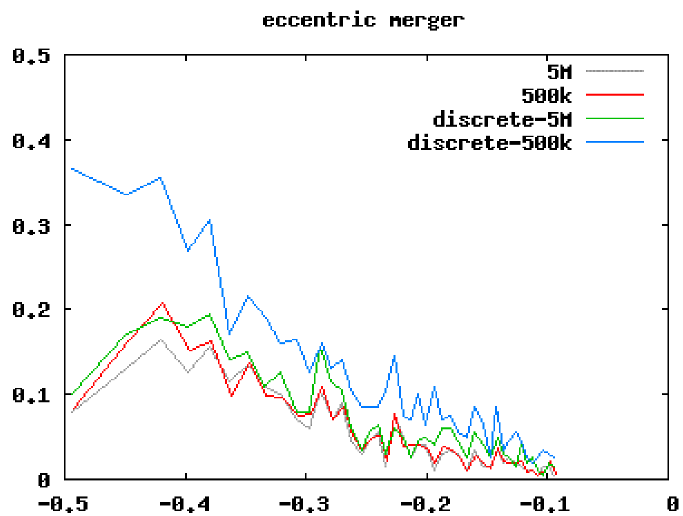


[Vasiliev & Athanassoula 2012]

Applications: centrophylic orbits and feeding rates of supermassive black holes

- Objective: study the rate at which stars arrive to the center of galaxy in a realistic galactic model (e.g. from a cosmological simulation, or from a merger of two galaxies) which is manifestly non-spherical
- Method: explore the properties of orbits in a frozen N-body potential or in a smooth spherical-harmonic approximation to the potential; find fraction of centrophylic orbits and the rate at which stars on these orbits approach to less than a given radius; extrapolate to N=∞

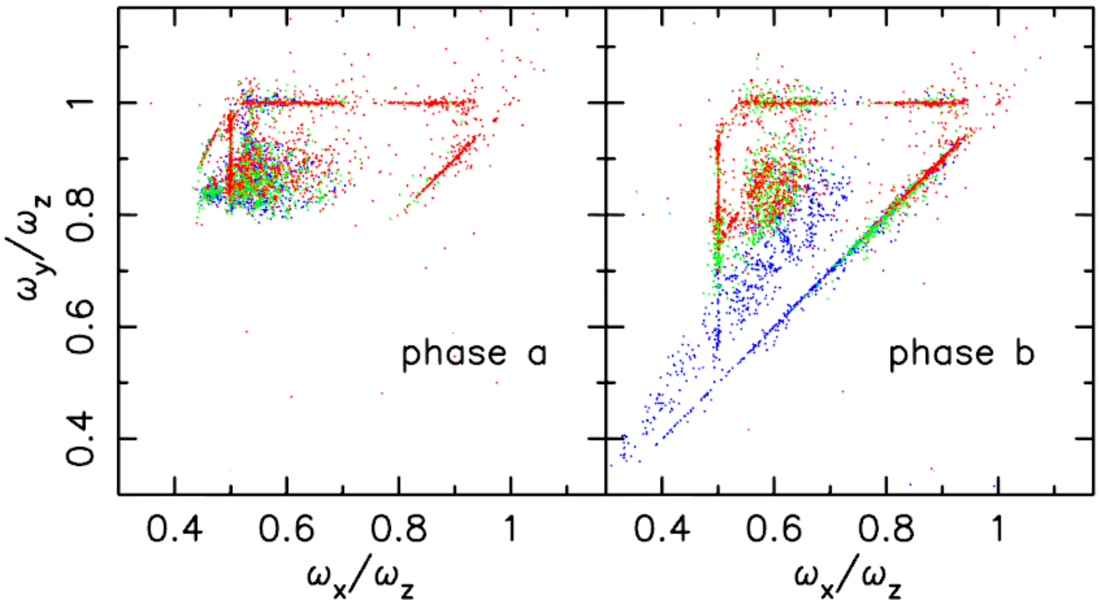




[Vasiliev, Antonini & Merritt, in prep.]

Applications: evolution of shape in galaxy formation and baryonic compression/expansion

- Study the causes and reversibility of shape evolution in the galaxy formation
- Explore orbital structure before and after “baryonic condensation”
- Find the conditions under which such evolution is reversible (i.e. shape is restored after “baryonic evaporation”); relate to the influence of chaotic orbits



[Valluri et al. 2009]

Conclusions

- Orbital analysis is a useful tool for studying the global properties of motion of stars in a galaxy model, including:
  - network of resonant orbits
  - chaotic properties of potential
  - abundance of centrophylic orbits
- In application to N-body simulations, one may use “frozen” N-body potential or a smooth approximation using variants of spherical-harmonic expansion
- Applications to many problems in galactic dynamics and cosmology