Supermassive black holes in non-spherical galactic nuclei and enhanced rates of star capture events

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Plan of the talk

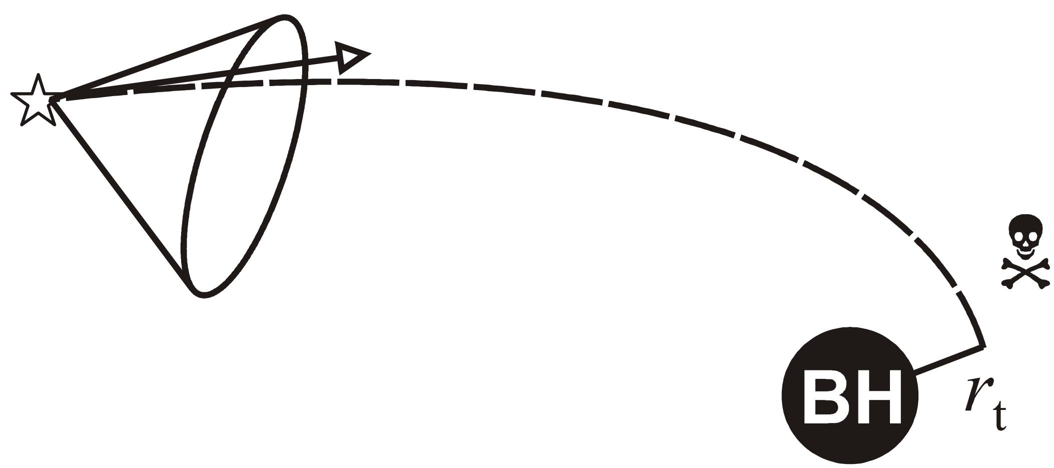
- Capture of stars by a supermassive black hole
- Orbits around black holes in non-spherical nuclei
- Difference between spherical, axisymmetric and triaxial nuclear star clusters
- Two-body relaxation in galactic nuclei
- Empty and full loss cone regimes
- Loss cone draining and refill rates
- Predictions for realistic galaxies; conclusions.

Capture of stars by a supermassive BH

The black hole captures or tidally disrupts stars passing at a distance closer than $r_t \geq r_\bullet \equiv \frac{2GM_\bullet}{c^2}$, or, equivalently, with angular momentum

$$L^2 < L_\bullet^2 = \max\left[\left(\frac{4GM_\bullet}{c}\right)^2, GM_\bullet r_t\right]$$

The region of phase space with $L < L_\bullet$ is called the loss cone.

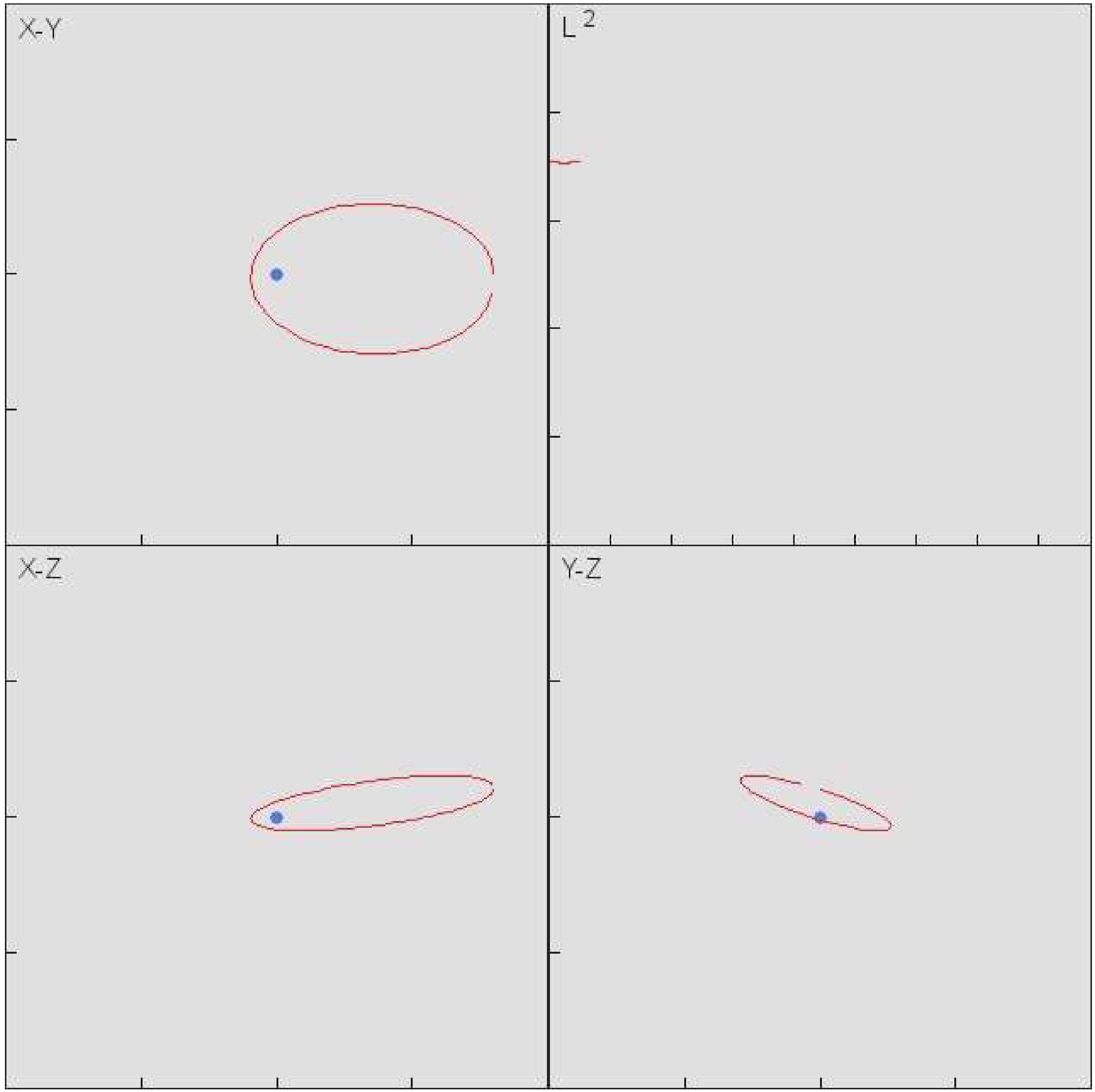


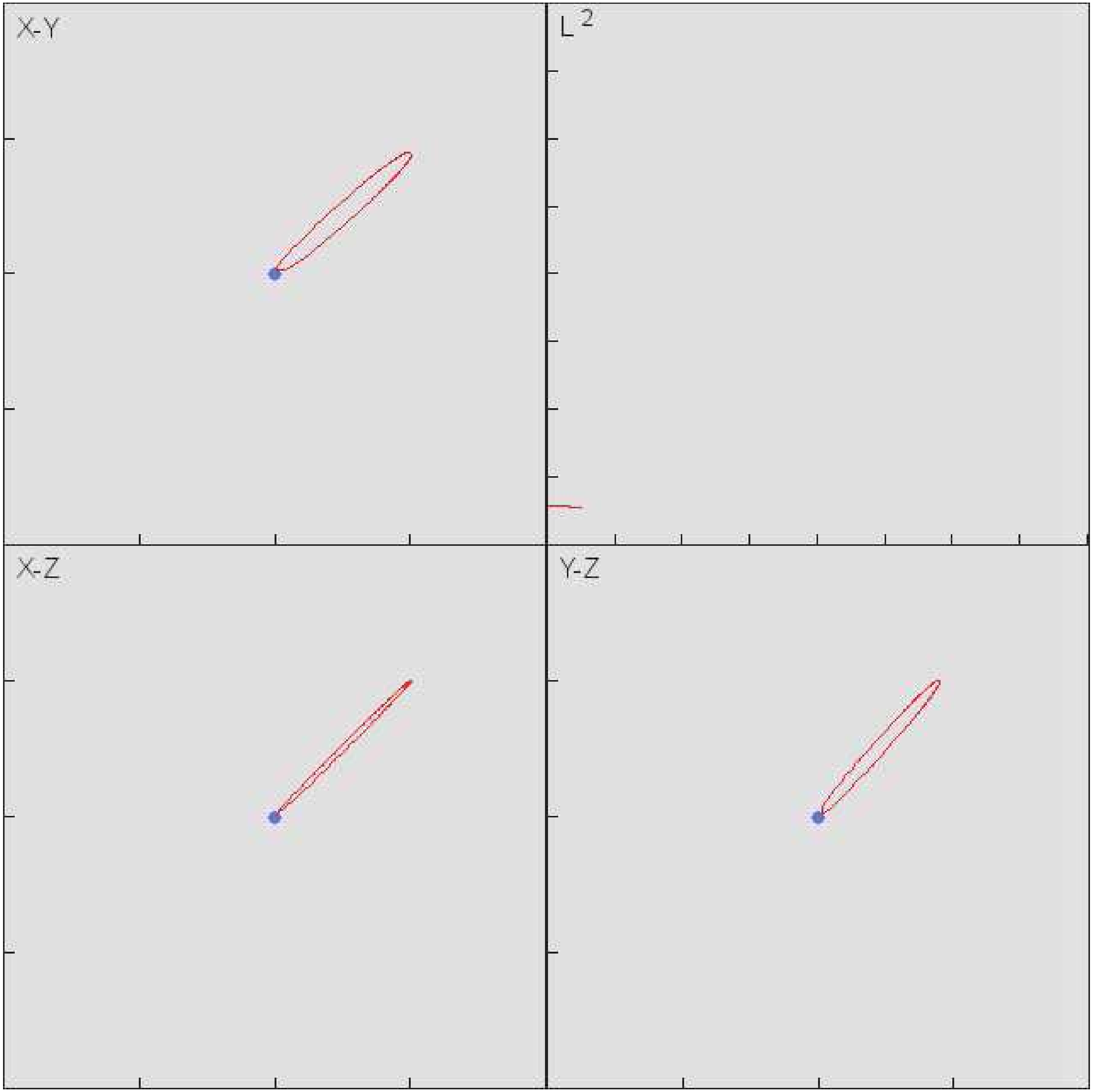
Nuclear star clusters

- Supermassive black hole M_{bh}
- Stellar cusp (for example, a power law density profile $\rho \sim r^{-\gamma}$)
- Total gravitational potential (non-spherical):

$$\Phi(\vec{r}) = -\frac{GM_{bh}}{r} + \Phi_\star \left(\frac{r}{r_0}\right)^{2-\gamma} \left(1 + \varepsilon \frac{z^2}{r^2} + \eta \frac{y^2}{r^2}\right)$$

- Consider motion inside radius of influence $r_{infl} = GM_{bh}/\sigma^2$
  => dominant contribution to potential is from SMBH
  => orbits are perturbed Keplerian ellipses
  which precess due to torques from stellar potential
- Orbital time $T_{rad} \ll$ precession time $T_{prec} \sim r_{infl}/\sigma$

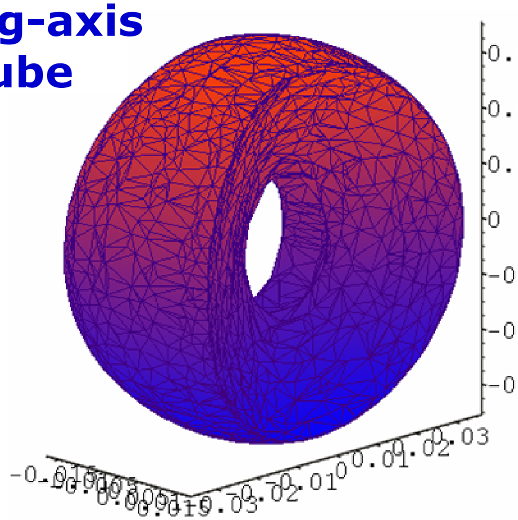

X-Y
L 2
X-Z
Y-Z


X-Y
L 2
X-Z
Y-Z

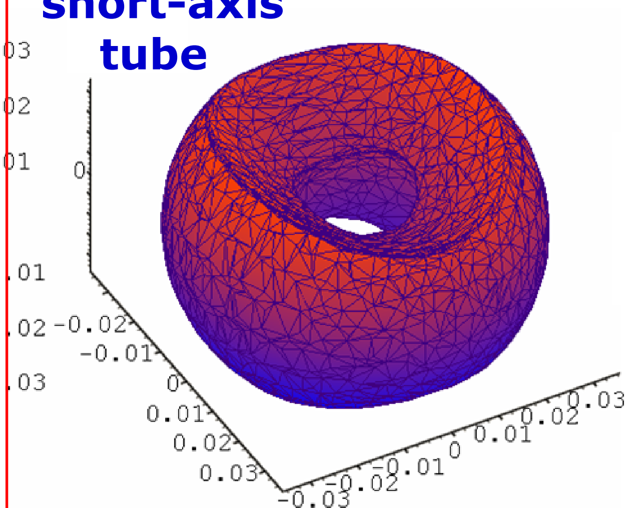
Types of orbits in non-spherical star cluster around a supermassive black hole

Triaxial cluster:

long-axis tube

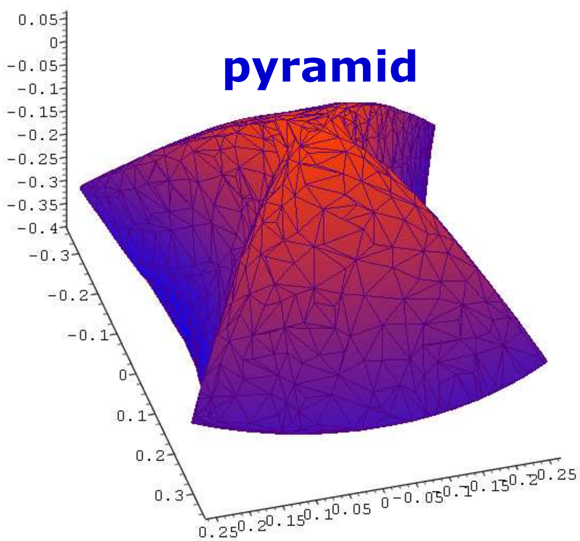


short-axis tube

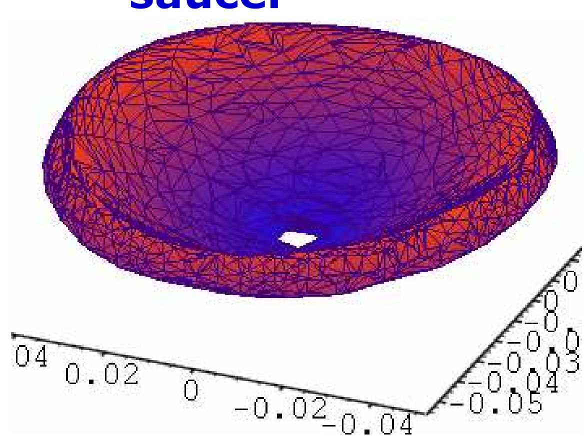


Axisymmetric cluster

pyramid



saucer



Motion in a near-keplerian potential

$$\Phi(\vec{r}) = -\frac{GM_{bh}}{r} + \Phi_\star \left(\frac{r}{r_0}\right)^{2-\gamma} \left(1 + \varepsilon \frac{z^2}{r^2} + \eta \frac{y^2}{r^2}\right)$$

- "Fast" timescale – radial period $T_{\rm rad} = \dfrac{2\pi\, r^3}{\sqrt{GM_\bullet}}$.
- "Slow" timescale – precession period due to distributed mass $T_{\rm prec} = T_{\rm rad} \dfrac{M_\bullet}{M_\star(r)}$.

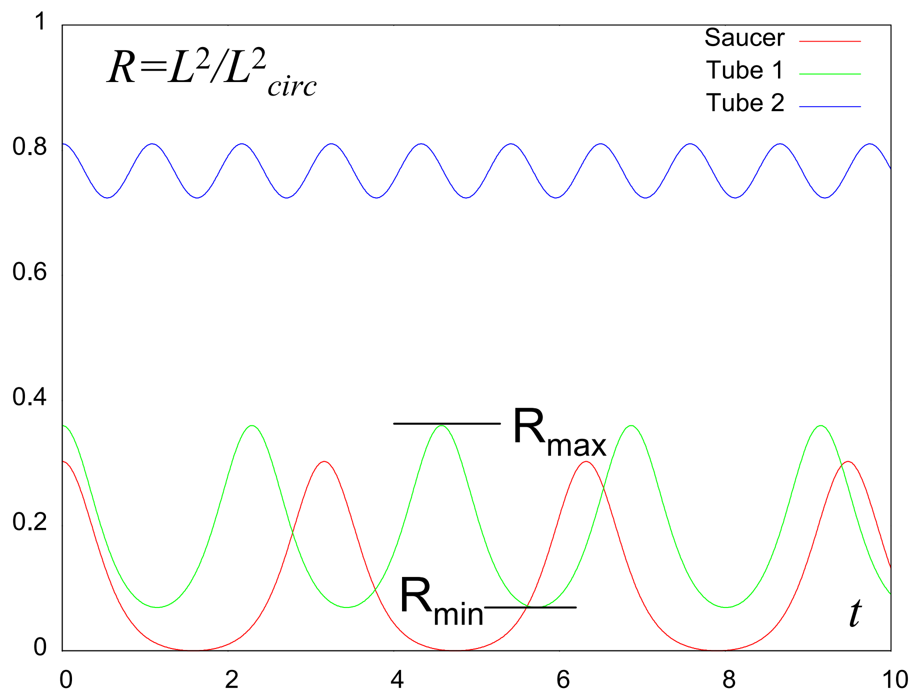
The separation of fast and slow timescales allows for the existence of an additional integral of motion $\mathcal{H} = \oint_{\rm orbit} \Phi_\star(r)$.

In both axisymmetric and triaxial cases the motion is **completely integrable**.

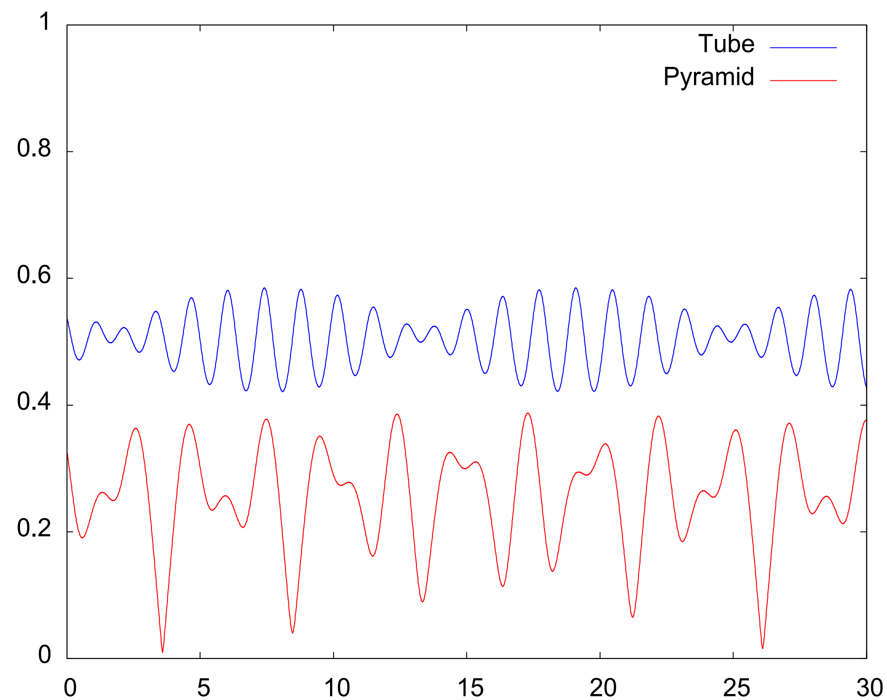
Integrals of motion: **E** (total energy), **H** (secular hamiltonian), $\mathbf{L_Z}$ (z-component of angular momentum) in axisymmetric case / another integral in triaxial case.

Evolution of angular momentum of an orbit in a non-spherical nuclear star cluster

Three integrals of motion: total energy **E**, secular hamiltonian **H**, and a third integral **W** which is reduced to z-component of angular momentum L_z in axisymmetric systems. Total angular momentum squared, $\mathbf{L^2}$, is not conserved but experiences oscillations between R_{min} and R_{max} with characteristic period ~ T_{prec}, and amplitude ~ ε.



Axisymmetric system



triaxial system

"Extended loss region"
in a non-spherical nuclear star cluster

The region of phase space (E, H, W) occupied by orbits for which
the squared angular momentum L^2 may drop below the capture boundary $L^2_\bullet$
is called **"extended loss region"**

- For axisymmetric systems, the condition of being in the extended loss region is $L_z < L_\bullet$ and $\langle L^2 \rangle < \varepsilon$ (i.e. only a fraction ~ ε of orbits with z-component of angular momentum below capture boundary may actually be captured).
- For triaxial systems, **all pyramid orbits**(*) are centrophylic (i.e. may attain arbitrary low values of angular momentum), their fraction in the total population is **~ ε**.

(*) relativistic effects change this conclusion for most tightly-bound orbits

Difference between spherical, axisymmetric and triaxial nuclear star clusters

	Spherical	Axisymmetric	Triaxial
Fraction of stars with $L^2_{min} < X$	$\propto X$	$\propto \sqrt{X\varepsilon}$	$\propto \varepsilon$
Fraction of time that such a star has $L^2 < X$ (i.e. capture probability)	1	$\sqrt{X}$	X
Survival time of such stars (assuming they are captured immediately after reaching $L^2 < R_{capt}$)	T_{rad} ($10^{1\text{-}5}$ yr)	T_{prec} ($10^{5\text{-}6}$ yr)	may be longer than 10^{10} yr

(for MW nucleus)

but that may not be true
in the presence of relaxation

fraction of stars
timescale for draining the loss region

increases

Two-body relaxation in galactic nuclei and the concept of empty/full loss cone

Relaxation time $T_{rel} = \frac{0.34\,\sigma(r)^3}{G^2\,\overline{m}_\star\,\rho_\star(r)\,\ln\Lambda}$ – timescale for diffusion in E and L

Definition of "classical" loss cone for the spherical case:

Loss cone is the region in phase space in which an orbit *would be captured*$^{(*)}$ at the nearest pericenter passage, i.e. at most within 1 radial period, having $L^2 < L^2_\bullet$

$^{(*)}$ in the absence of relaxation

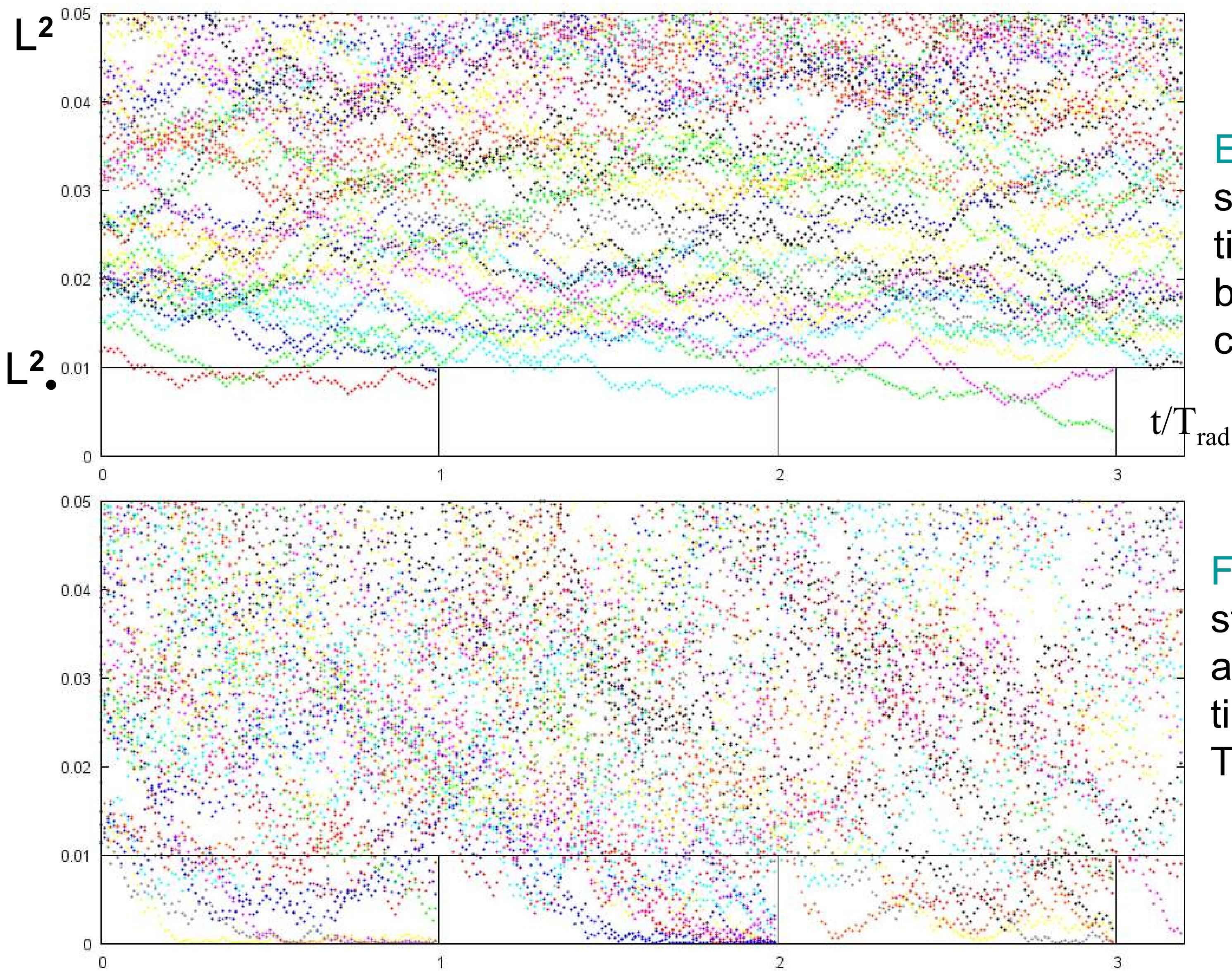
The question is how fast the changes in L occur compared to radial period:

$q = \Delta L^2/L^2_\bullet$

q << 1 – empty loss cone regime:
stars are captured as soon as they enter LC; population of stars with $L<L_\bullet$ is small

q >> 1 – full loss cone:
stars may move in and out of LC many times before being captured at the end of T_{rad}, distribution function of stars inside LC is the same as outside (near its boundary)

The concept of empty/full loss cone

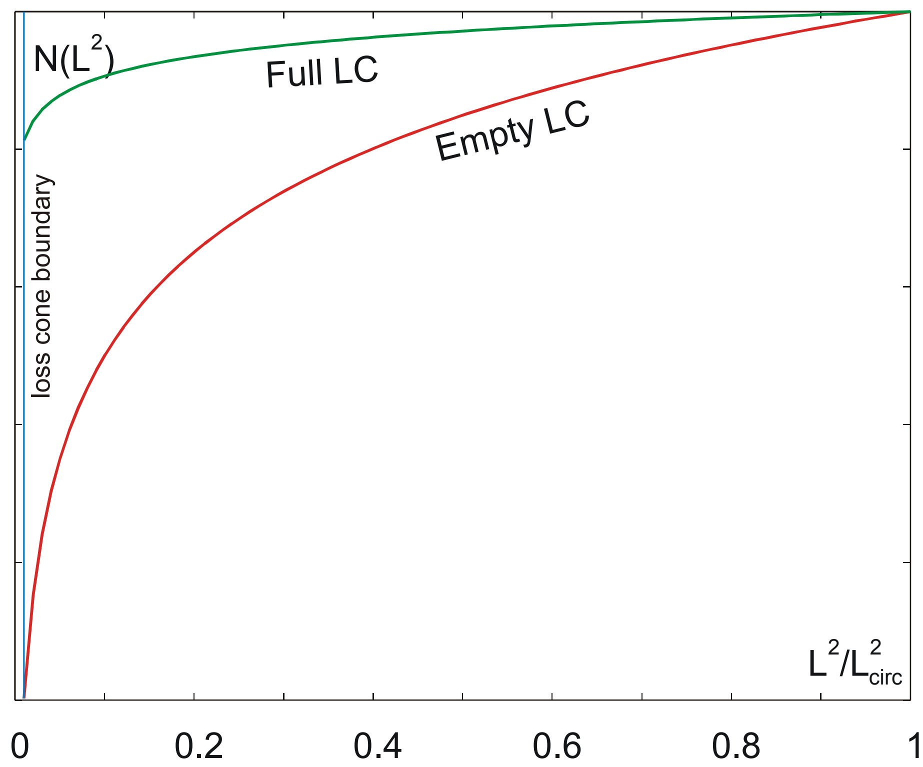


Empty LC:
stars barely have time to enter LC before they get captured after T_{rad}

Full LC:
stars may enter and exit LC many times during one T_{rad}

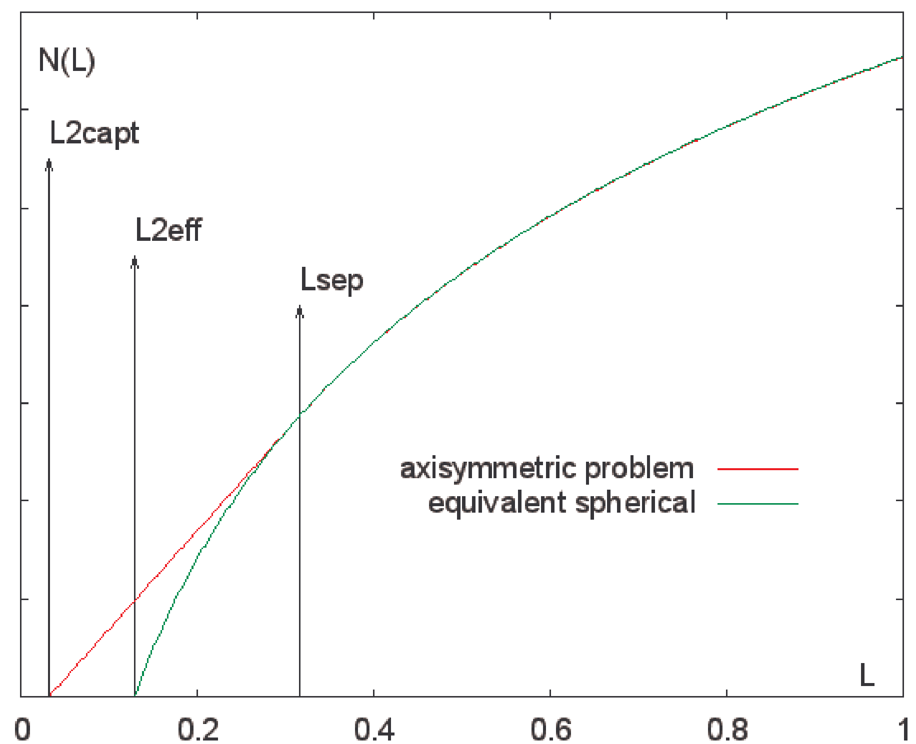
Empty/full loss cone regimes in spherical galaxies

- Distribution function of stars in angular momentum is a solution to Fokker-Planck (diffusion) equation with a “sink” at loss cone boundary
- The global quasi-stationary solution at given E has a logarithmic profile: $N(L^2) = N_0 + A*\log(L)$
- In the empty loss cone regime, the capture rate is dominated by the slope of N(L), i.e. limited by the diffusion coefficient
- In the full loss cone regime, capture rate is simply the size of loss cone divided by T_{rad}
- Stars close to BH are in empty regime, outer parts of the galaxy may be in full loss cone regime



Non-spherical galaxies:
Loss cone draining vs. relaxation

- Draining time of the loss region may be $>10^{10}$yr in the triaxial case, and the capture rate from the loss region depends on the efficiency of changing of ang.momentum due to regular precession rather than due to 2-body relaxation.
- After all orbits with $L^2_{min}<L^2_\bullet$ have been drained, the influx of stars from higher L is still limited by diffusion (relaxation)
- Because the size of loss region is larger that spherical loss cone, flux will be larger in the diffusion-limited regime
- In the full loss cone regime there is almost no difference from the spherical case



Conclusions

- In non-spherical nuclear star clusters the star angular momentum L is changed not only due to 2-body relaxation, but also due to regular precession in the smooth additional potential of stellar cluster
- This facilitates the capture of stars at low L:
  the "extended loss region" is where $L^2_{min}<L^2_\bullet$, not just $L^2<L^2_\bullet$
- Draining time of this region $T_{drain} \sim T_{prec} \sim 10^{5\text{-}6}$ yr in axisymmetric case and much longer, comparable to Hubble time, in triaxial case
- For $T \gg T_{drain}$, capture rate is determined by relaxation with a larger effective capture boundary. Compared to the spherical case, the enhancement in total capture rate is moderate (~factor of few) and is important only in the range of energies where the loss cone would be empty in the spherical case.
- For giant elliptical galaxies, which are deeply in the empty loss cone regime, the enhancement may be more dramatic.
- This applies to the rates of tidal disruption events, EMRI, binary SMBH, …