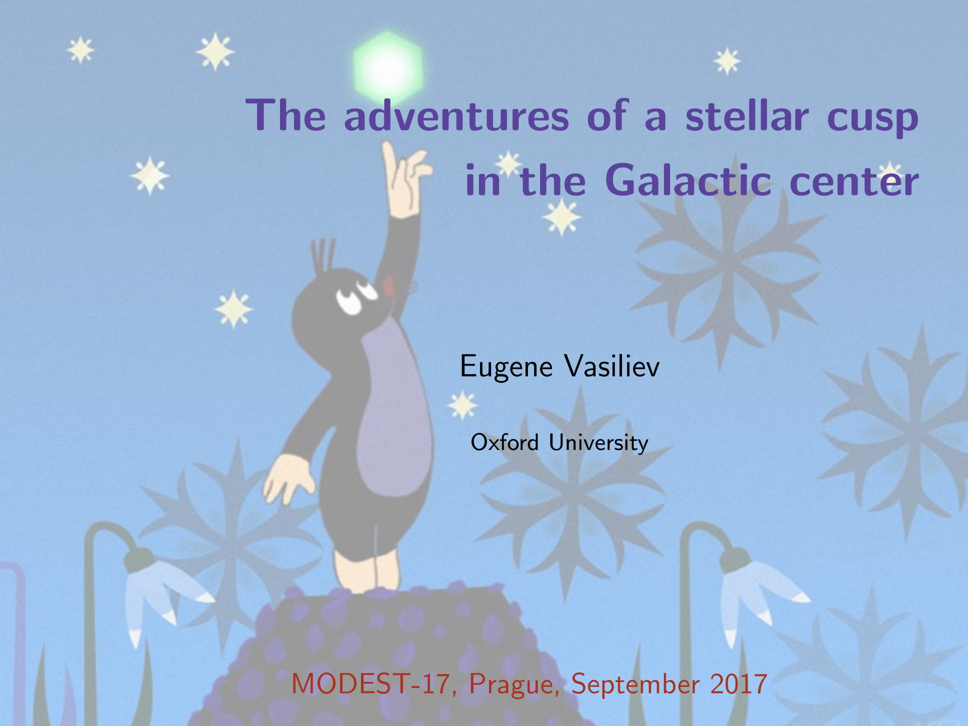

The adventures of a stellar cusp in the Galactic center

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Overview

Goal:

study the evolution of self-gravitating stellar systems driven by two-body relaxation.

Methods:

complexity

- semianalytic, scaling models [Hénon 1967,75; Gieles+ 2014]
- fluid models [Lynden-Bell&Eggleton 1980; Louis&Spurzem 1991]
- Fokker–Planck models [Cohn 1980s; Takahashi 1990s; Spurzem+ 2000s]
- Monte Carlo methods [Spitzer+ 1970s; Hénon 1971; Marchant&Shapiro 1979; Giersz 1998; Joshi+ 2000; Freitag&Benz 2002; Vasiliev 2015]
- N-body simulations [Aa 1960s–. . . , B, C, D, H&H, . . .]

Fundamentals of Fokker–Planck models

- Stellar system is described in terms of a smooth potential $\Phi(\mathbf{x}; t)$ and a smooth distribution function $f(E\,[, L, \dots]; t)$.
- Evolution is slow (compared to dynamical time) $\Longrightarrow$ use the orbit-averaged approximation (all quantities may depend only on the integrals of motion and time).
- Two-body relaxation (large number of uncorrelated encounters) leads to the diffusion of f in the space of integrals of motion.
- Advection and diffusion coefficients depend on f themselves (non-linear parabolic PDE for f).
- Potential is recomputed from DF: first obtain the density $\rho(\mathbf{x}) = \int f(\dots)\,\mathrm{d}v$, then solve the Poisson equation (integro-differential equation for $\{f, \Phi\}$).

Additional assumptions in my implementation

- Spherical symmetry.
- Isotropic distribution function $f(E)$.
- Standard ("Chandrasekhar/Spitzer") relaxation prescription.

Motivation

- Captures essential thermodynamical evolution.
- Clean, noise-free laboratory.
- Enough flexibility for many situations
  (mass spectrum, star formation, loss cone, ...)
- Fast! (few seconds to minutes per run).

Mathematical details

Phase volume h instead of energy E as the independent variable:

$$
\begin{aligned}
h(E) &\equiv \iiint \mathrm{d}^3x \iiint \mathrm{d}^3v \begin{cases} 1 & \text{if } \Phi(|\mathbf{x}|) + \frac{1}{2}|\mathbf{v}|^2 < E \\ 0 & \text{otherwise} \end{cases} \\
&= \frac{16\pi^2}{3} \int_0^{r_\mathrm{max}(E)} r^2 \left\{ 2\big[E - \Phi(r)\big] \right\}^{3/2} \mathrm{d}r \\
&= 4\pi^2 \int_0^{L^2_\mathrm{circ}(E)} J_r(E, L)\, \mathrm{d}L^2, \quad \text{where } J_r \text{ is the radial action} \\
&= \int_{\Phi(0)}^{E} g(E)\, \mathrm{d}E, \quad \text{where } g(E) \text{ is the density of states}
\end{aligned}
$$

- Mass of stars in the interval $\mathrm{d}h$ is $\mathrm{d}M = f(h)\,\mathrm{d}h$
- DF is conserved under adiabatic change of potential
  (when updating the potential via Poisson equation, $f(h)$ does not change)

Mathematical details

Flux-conservative form of the Fokker–Planck equation:

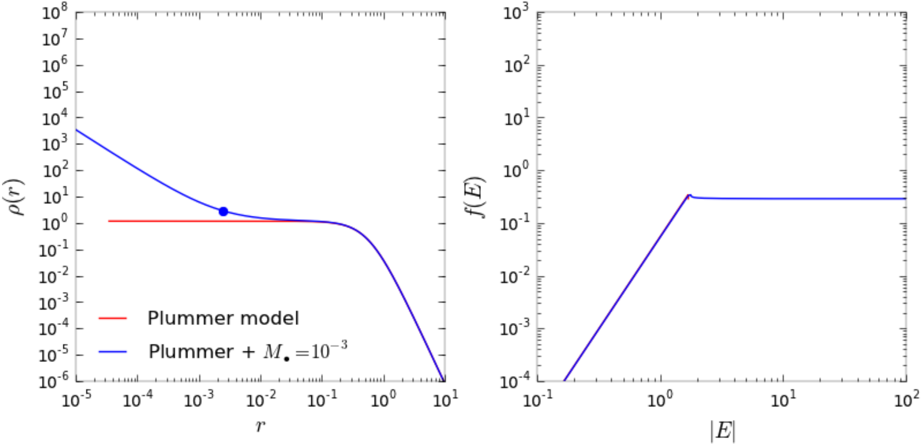
$$\frac{\partial f(h,t)}{\partial t} = \frac{\partial}{\partial h}\Bigg[\overbrace{\underbrace{A\{f\}\, f(h,t)}_{\text{advection}} + \underbrace{D\{f\}\, \frac{\partial f(h,t)}{\partial h}}_{\text{diffusion}}}^{\text{flux in phase space}}\Bigg] + \underbrace{s(h,t)}_{\text{source}} - \underbrace{\nu\, f(h,t)}_{\text{sink}}$$

$$A = 16\pi^2 G^2\, m_\star \ln\Lambda \int_0^h \mathrm{d}h'\; f(h')$$

$$D = 16\pi^2 G^2\, m_\star \ln\Lambda \int_0^\infty \mathrm{d}h'\; f(h')\; \min(h,h')\; \frac{g(h)}{g(h')}$$

High-accuracy finite-element method for discretized PDE.

The rise and fall of a Bahcall–Wolf cusp



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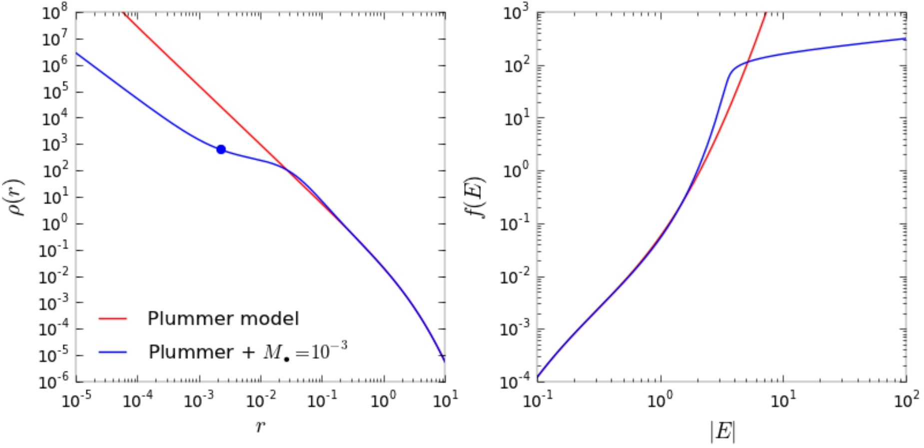
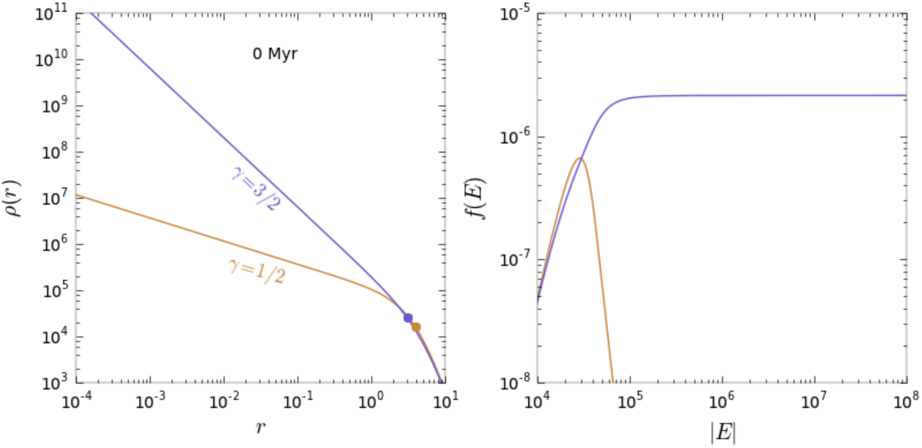


Interesting facts about the Bahcall–Wolf solution

- The density distribution around a central black hole forms a cuspy profile $\rho \propto r^{-7/4}$, while the DF is $f(E) \propto |E|^{1/4}$.
- However, its amplitude does not stay constant, but evolves with time (mass flux is very small but can have either sign).
- Black hole acts as a source of energy, heating up the stellar system (energy flux is finite and always directed away from the BH).
- Energy is transported by conduction, not advection.
- Heating rate does not depend on whether stars are consumed by the BH or not, and is determined by the maximum thermal conductivity of the stellar system.
- At late times, the system expands self-similarly: $r(t) \propto t^{-2/3}$ [e.g. Hénon 1975].

[Re-]growth of a Bahcall–Wolf cusp

Milky Way nucleus: $M_\bullet = 4 \times 10^6\, M_\odot$; initial profile: $\rho \propto r^{-\gamma}$, with $\gamma = 1/2$ "core" [Merritt 2010] or $\gamma = 3/2$ "cusp".



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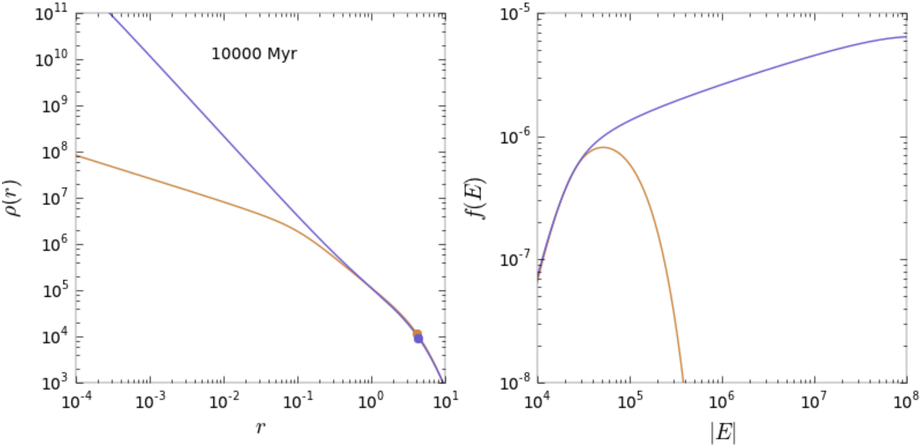
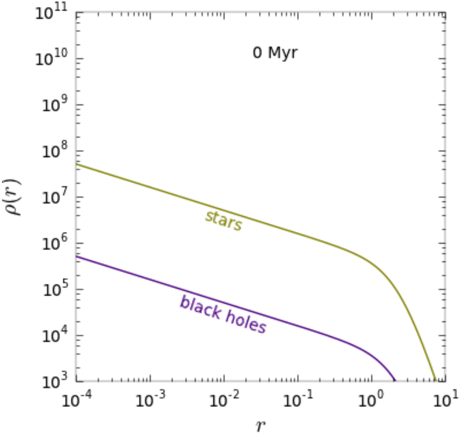
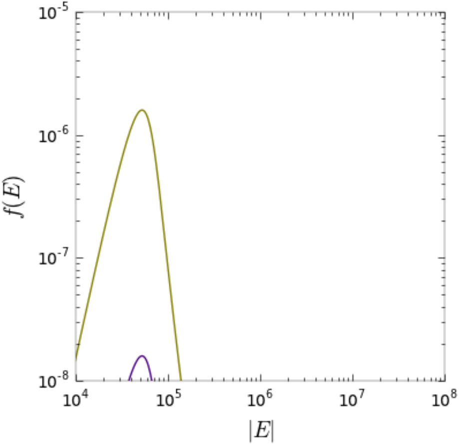


Two-component model with mass segregation

Stars: $m_1 = 1\,M_\odot$; stellar black holes (1% by mass): $m_2 = 10\,M_\odot$.
initial profile: $\gamma = 1/2$ "core"





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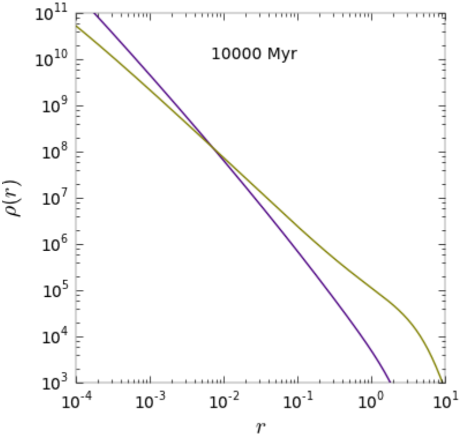
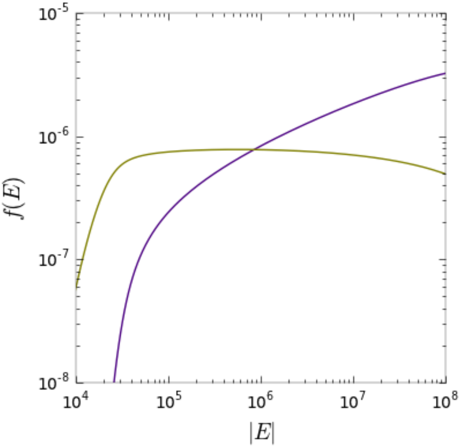
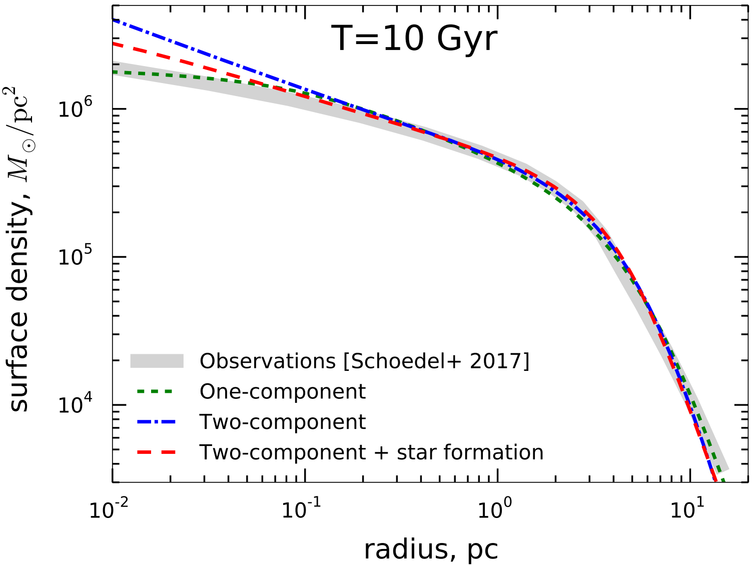




Models of the Milky Way nuclear star cluster



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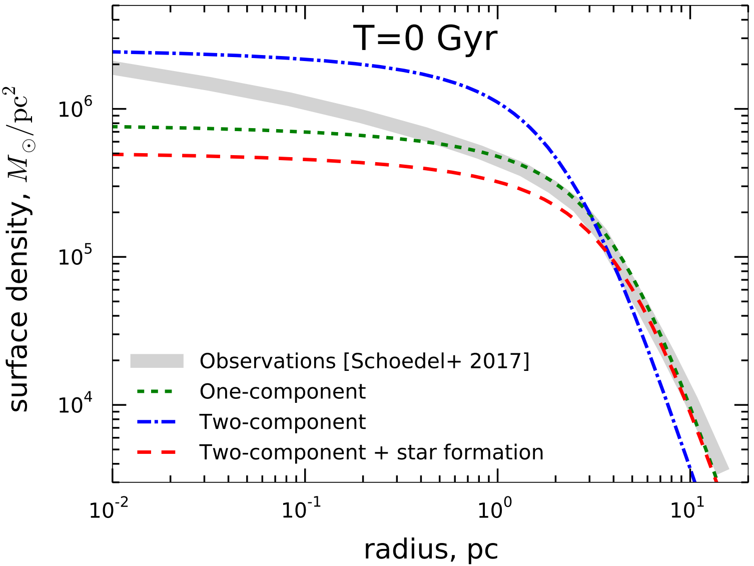

Summary

- Fokker–Planck approach is still useful
- Bahcall–Wolf cusp is a live creature
- Thermodynamical evolution is important
- The code is available for the community:

  https://td.lpi.ru/~eugvas/phaseflow

  see https://arxiv.org/abs/1709.04467