A new Monte-Carlo method for dynamical evolution of non-spherical stellar systems

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Overview

- Motivation
- Method
- Tests
- Applications

Motivation

Galactic nuclei with supermassive black holes:

- The rate of capture of stars by the black hole depends on the efficiency of angular momentum variation of individual stars;
- Angular momentum changes both due to two-body relaxation (collisional) and because of torques in a non-spherical potential (collisionless);
- The number of stars in realistic galactic nucleus far exceeds the presently accessible range for collisional N-body simulations;
- Scaling to a different number of particles would distort the interplay between collisional and collisionless effects;
  Need to adjust the relaxation rate independently from $N_{particles}$;
- Fokker-Planck and fluid models are impractical for complex geometry
  => need to use a particle-based Monte-Carlo method.

Monte-Carlo methods

Table 1. Comparison of Monte Carlo methods

Name	Reference	relaxation treatment	timestep	1:1[a]	BH[b]	remarks
Princeton	Spitzer & Hart (1971); Spitzer & Thuan (1972)	local dif.coefs. in velocity, Maxwellian background $f(r, v)$	$\propto T_{\rm dyn}$	no	no	
Cornell	Marchant & Shapiro (1980)	dif.coef. in E, L, self-consistent background $f(E)$	indiv., $T_{\rm dyn}$	no	yes	particle cloning
Hénon	Hénon (1971a)	local pairwise interaction, self-consistent bkgr. $f(r, v_{\|\|}, v_{\perp})$	$\propto T_{\rm rel}$	no	no	
	Stodółkiewicz (1982) Stodółkiewicz (1986)	Hénon's	block, $T_{\rm rel}(r)$	no	no	mass spectrum, disc shocks binaries, stellar evolution
Mocca	Giersz (1998)	same	same	yes	no	3-body scattering (analyt.)
	Hypki & Giersz (2013)	same	same	yes	no	single/binary stellar evol., few-body scattering (num.)
Cmc	Joshi et al. (2000)	same	$\propto T_{\rm rel}$(center)	yes	no	partially parallelized
	Umbreit et al. (2012), Pattabiraman+ (2013)		(shared)	yes	yes	fewbody interaction, single/binary stellar evol., GPU
Me(ssy)[2]	Freitag & Benz (2002)	same	indiv.$\propto T_{\rm rel}$	no	yes	cloning, SPH physical collis.
Raga	this study	local dif.coef. in velocity, self-consistent background $f(E)$	indiv.$\propto T_{\rm dyn}$	no	yes	arbitrary geometry

[a] One-to-one correspondence between particles and stars in the system
[b] Massive black hole in the center, loss-cone effects

Spitzer's Monte-Carlo method

Local (position-dependent) velocity diffusion coefficients:

$$v\langle\Delta v_\parallel\rangle = -\left(1+\frac{m}{m_\star}\right) I_{1/2}\,,$$
$$\langle\Delta v_\parallel^2\rangle = \frac{2}{3}\left(I_0 + I_{3/2}\right),$$
$$\langle\Delta v_\perp^2\rangle = \frac{2}{3}\left(2I_0 + 3I_{1/2} - I_{3/2}\right),$$

Particles move in a given smooth potential with arbitrary geometry, and velocity perturbations are applied according to 2-body relaxation theory

here m and $m_\star$ are masses of the test and field stars, and

$$I_0 \equiv \Gamma \int_E^0 dE' \boxed{f(E'),}$$

Distribution function of field stars (isotropic bkg. approx.)

$$I_{n/2} \equiv \Gamma \int_{\Phi(r)}^E dE'\, f(E') \left(\frac{E' - \boxed{\Phi(r)}}{E - \Phi(r)}\right)^{n/2},$$

Gravitational potential

$$\Gamma \equiv 16\pi^2 G^2 m_\star \ln\Lambda = 16\pi^2 G^2 M_{\rm tot} \times \boxed{(N_\star^{-1}\ln\Lambda).}$$

Scalable amplitude of perturbation

After each timestep, the perturbations to the velocity are computed as

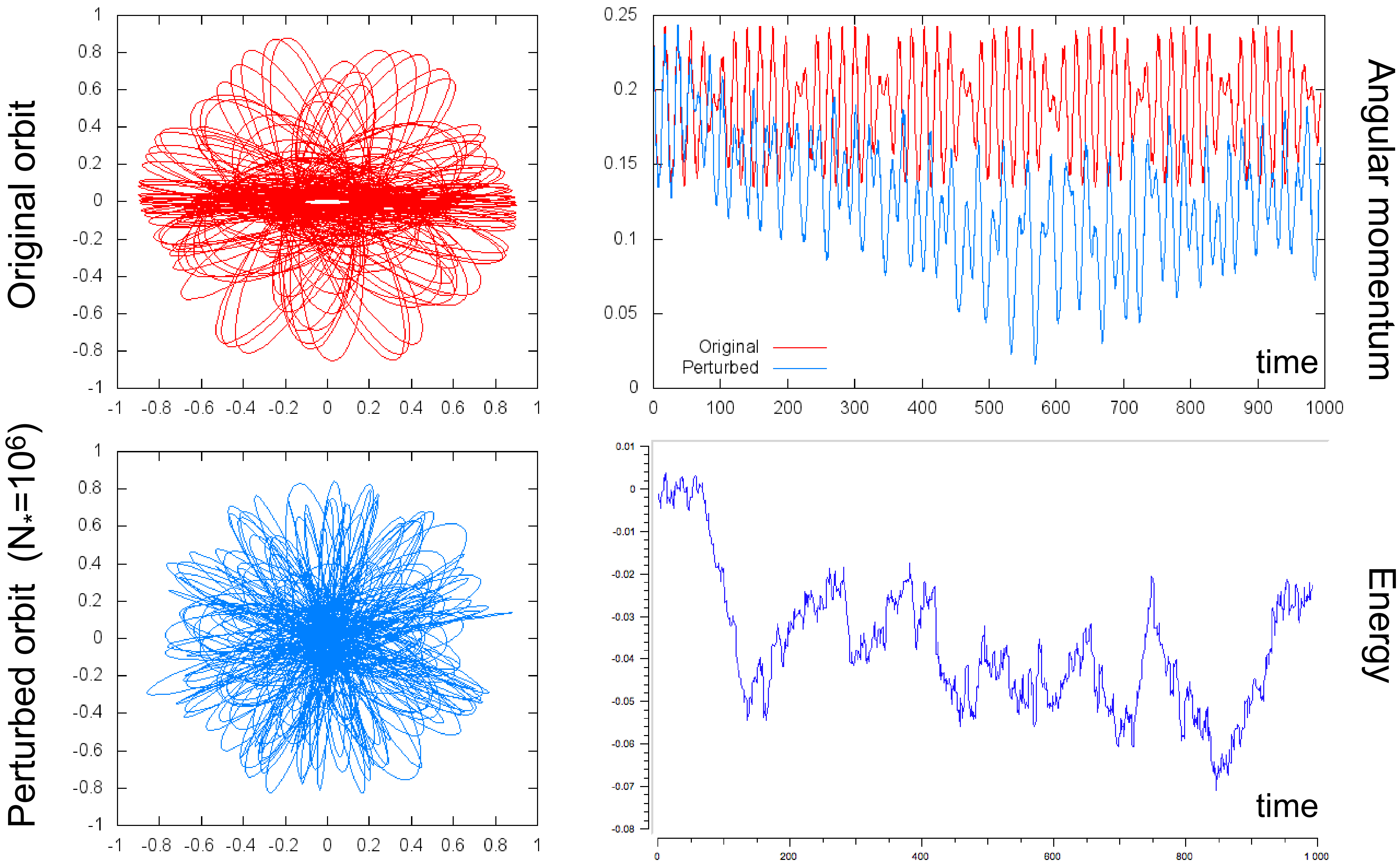
$$\Delta v_\parallel = \langle\Delta v_\parallel\rangle\Delta t + \zeta_1\sqrt{\langle\Delta v_\parallel^2\rangle\Delta t}\,,$$
$$\Delta v_\perp = \zeta_2\sqrt{\langle\Delta v_\parallel^2\rangle\Delta t}\,,$$

where ζ_1, ζ_2 are two independent normally distributed random numbers.

The new Monte-Carlo method

- Potential representation:
  basis-set expansion in spherical harmonics and radius (like SCF);
  adaptable to any geometry (with a well-defined center)
- Orbit integration:
  variable timestep Runge-Kutta;
  orbits are computed in parallel, independently from each other
- Relaxation:
  diffusion coefficients computed under an approximation of
  a spherical isotropic distribution function of background stars
- Potential and DF update:
  update interval >> dynamical time => temporal smoothing;
  each orbit is sampled with many points during update interval =>
  reduced discreteness noise

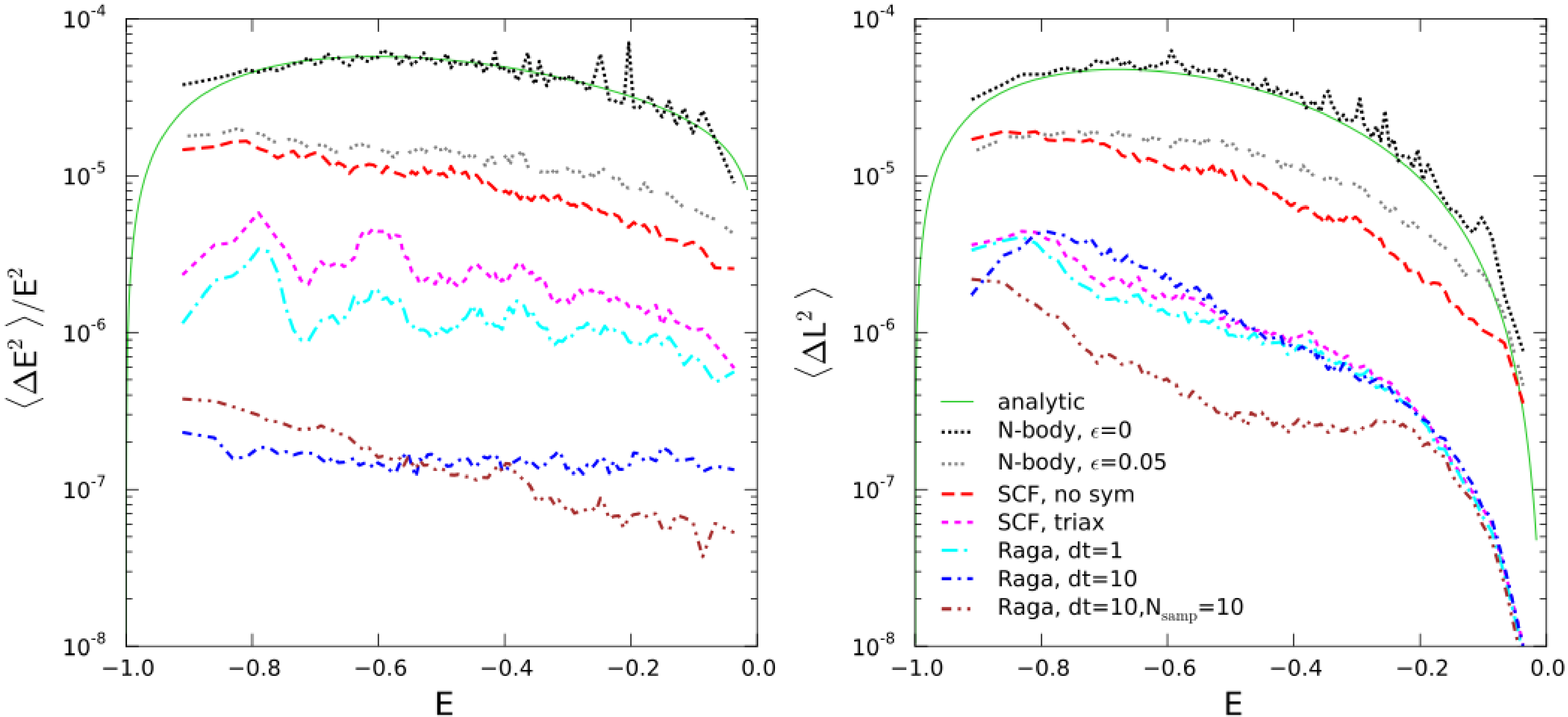
An example of orbits in a triaxial potential



Suppression of unwanted two-body relaxation

Test: a Plummer sphere with 10^5 particles;
measure the energy and angular momentum relaxation rate as functions of energy.

Temporal smoothing reduces the relaxation rate by 2 orders of magnitude!

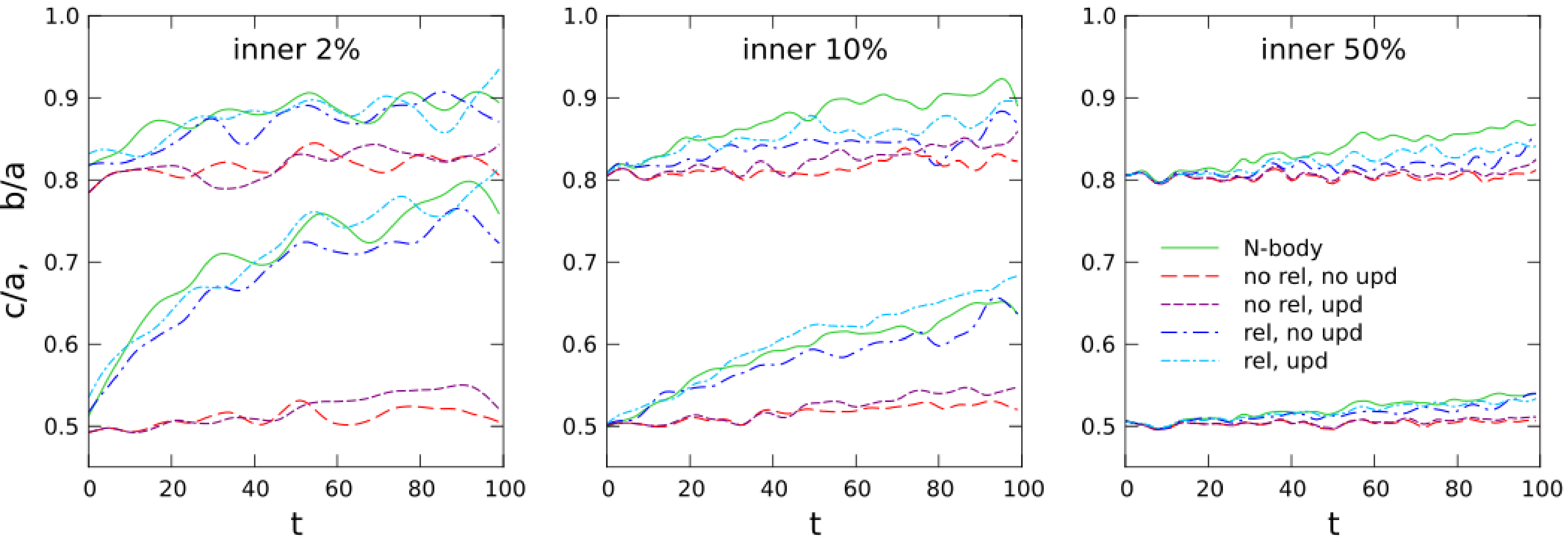


Suppression of shape evolution

Test: a triaxial Hernquist sphere with 10^5 particles and a central black hole $M_\bullet=0.01$; measure the evolution of model shape as a function of time (should stay constant?)

Fluctuations from 2-body interactions enhance the diffusion of chaotic orbits and lead to the decrease of triaxiality in the central parts of the model.

Switching off the relaxation greatly slows down the evolution of shape.

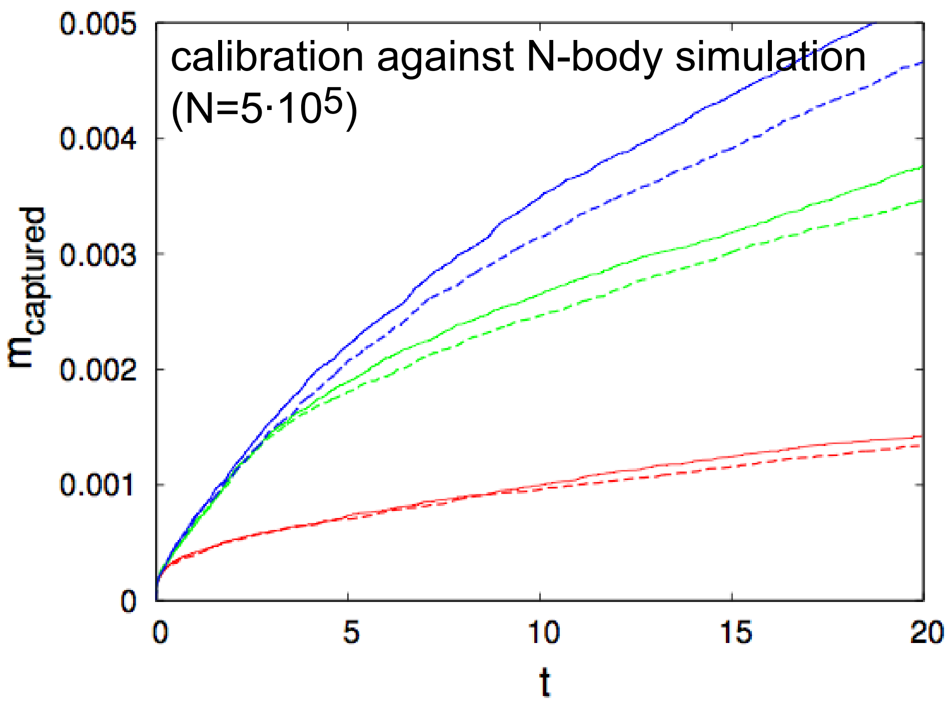


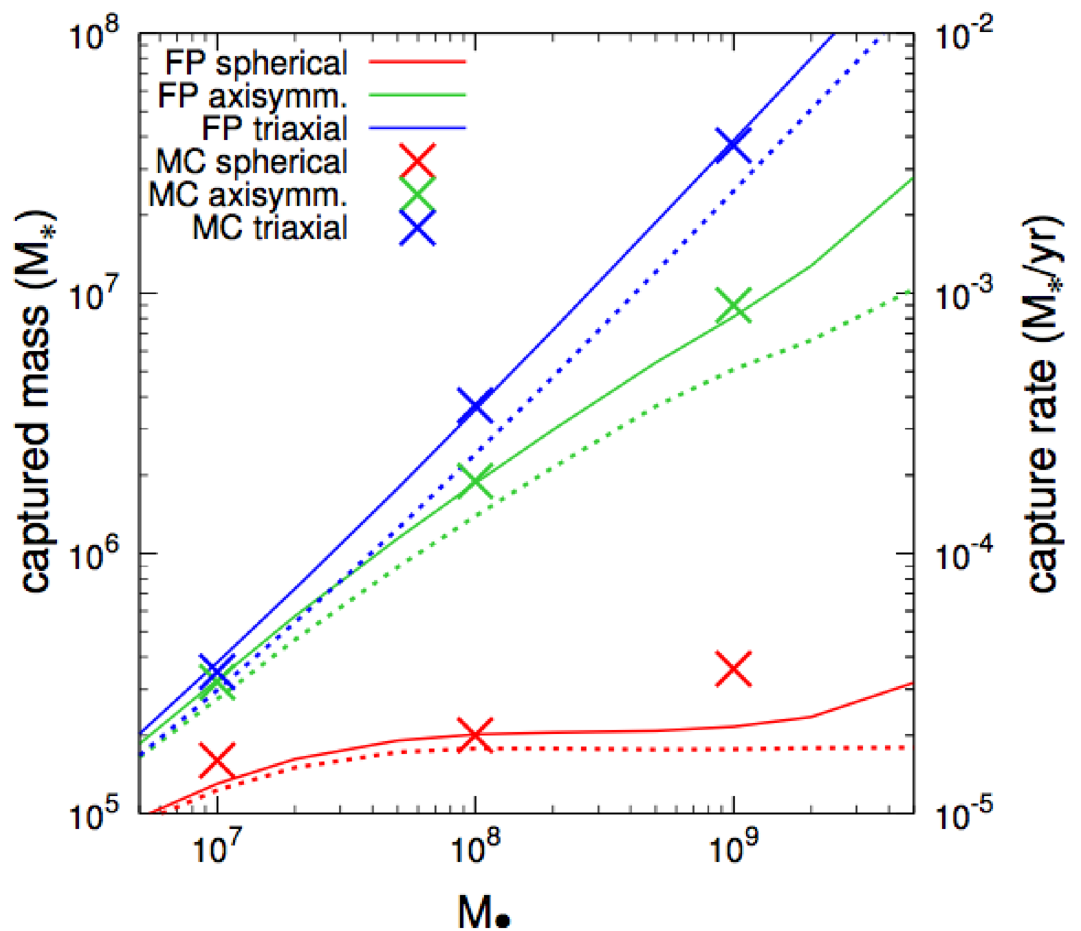
Application to black hole feeding rates

Compute the capture rates of stars in spherical, axisymmetric, and triaxial galactic nuclei.

For $M_\bullet > 10^7\ M_\odot$, the two-body relaxation time is much longer than Hubble time, while in a non-spherical potential the angular momentum variations are much greater and lead to substantially higher rate of capture of stars by the supermassive black hole.

This is a problem presently inaccessible with direct N-body simulations! ($N>10^9$)





Limitations: (* – work in progress)

- No mass spectrum (*), no stellar evolution (*), no primordial/dynamically formed binaries, no stellar collisions
- No exact energy conservation (a correction applied after each update step)
- Assuming isotropic spherical background in computing diffusion coefs

Applications and future prospects:

- Black hole feeding rates with realistic mass spectrum and stellar evolution (*)
- Binary supermassive black holes and the final-parsec problem in non-spherical galaxies (*)
- Influence of discreteness noise on the diffusion of chaotic orbits
- Dynamical friction in non-spherical stellar systems
- Rotating flattened globular clusters
- …?