Evolution of binary supermassive black holes and the final parsec problem

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Plan of the talk

Why binary supermassive black holes?

Evolutionary stages of binary black holes

Loss cone theory in spherical and non-spherical systems

The novel Monte Carlo simulation method

Results and conclusions

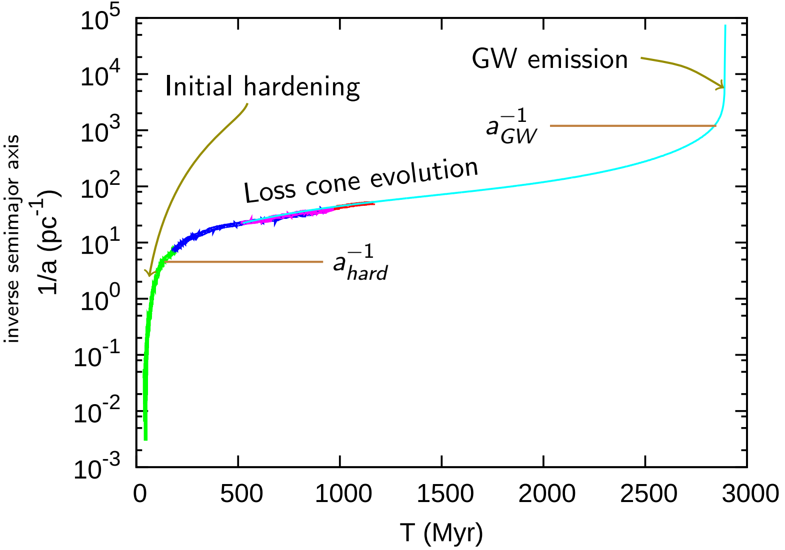
Origin of binary supermassive black holes (SBH)

- Most galaxies are believed to host central massive black holes
- In the hierarchical merger paradigm, galaxies in the Universe have typically 1–3 major and multiple minor mergers in their lifetime
- Every such merger brings two central black holes from parent galaxies together to form a binary system
- We dont see much evidence for widespread binary SBH (to say the least) – therefore they need to merge rather efficiently
- Merger is a natural way of producing huge black holes from smaller seeds

Evolutionary track of binary SBH

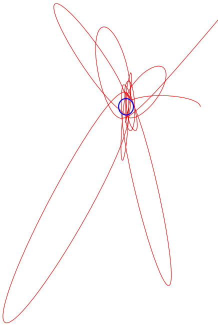
- Merger of two galaxies creates a common nucleus; dynamical friction rapidly brings two black holes together to form a binary (distance: $a \sim 10$ pc)
- Three-body interaction of binary with stars of galactic nucleus ejects most stars from the vicinity of the binary by the slingshot effect; a "mass deficit" is created and the binary becomes "hard" ($a \sim 1$ pc)
- The binary further shrinks by scattering off stars that continue to flow into the "loss cone", due to two-body relaxation or other factors
- As the separation reaches $\sim 10^{-2}$ pc, gravitational wave emission becomes the dominant mechanism that carries away the energy
- Reaching a few Schwarzschild radii ($\sim 10^{-5}$ pc), the binary finally merges

Evolutionary stages and timescales



[from Khan+ 2012]

Gravitational slingshot and binary hardening



A star passing at a distance $\lesssim 3a$ from the binary will experience a complex 3-body interaction which results in ejection of the star with velocity $v_{\rm ej} \sim \sqrt{\dfrac{m_1 m_2}{(m_1+m_2)^2}}\, v_{\rm bin}$.
These stars carry away energy and angular momentum from the binary, so that its semimajor axis a decreases:

$$\frac{d}{dt}\left(\frac{1}{a}\right) \approx 16\frac{G\,\rho}{\sigma} \;\equiv\; H_{\rm full}$$ [Quinlan 1996]

Thus, if density of field stars ρ remains constant, the binary hardens with a constant rate.
However, the reservoir of low angular momentum stars which can be ejected is finite and may be depleted quickly, so that the binary stalls at a radius $a_{\rm stall} \sim (0.1-0.4)a_h$.

Loss cone theory

The region of phase space with angular momentum $L^2 < L^2_{\mathrm{LC}} \equiv 2G(m_1 + m_2)\, a$ is called the loss cone. Gravitational slingshot eliminates stars from the loss cone in one orbital period T_{orb}. The crucial parameter for the evolution is the timescale for repopulation of the loss cone.

In the absence of other processes, the repopulation time is

$$T_{\mathrm{rep}} \sim T_{\mathrm{rel}} \frac{L^2_{\mathrm{LC}}}{L^2_{\mathrm{circ}}}, \text{ where } T_{\mathrm{rel}} = \frac{0.34\, \sigma^3}{G^2\, m_\star\, \rho_\star\, \ln \Lambda} \text{ is the relaxation time.}$$

If $T_{\mathrm{rep}} \lesssim T_{\mathrm{orb}}$, the loss cone is full (refilled faster than orbital period). In real galaxies, however, the opposite regime applies – the empty loss cone. In this case the hardening rate

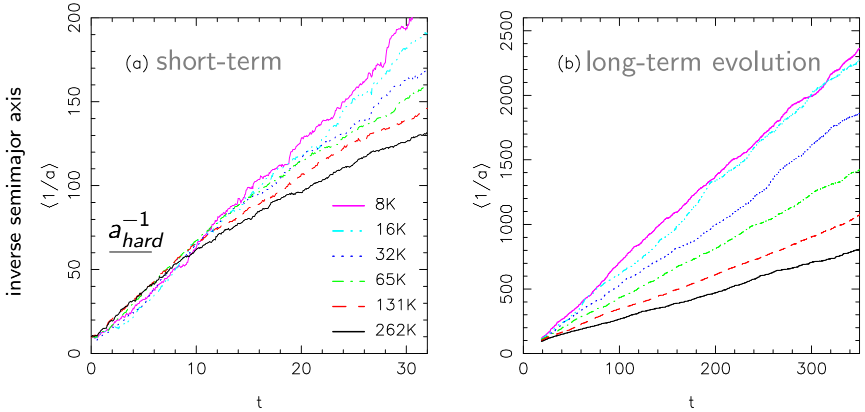
$$H \equiv \frac{d}{dt}(a^{-1}) \simeq \frac{T_{\mathrm{orb}}}{T_{\mathrm{rep}}} H_{\mathrm{full}}.$$

Relaxation is too slow for an efficient repopulation of the loss cone: in the absense of other processes the binary would not merge in a Hubble time.

This is the "final parsec problem" [Milosavljević&Merritt 2003]

N-scaling in the empty loss cone regime

Hardening rate $H \equiv \frac{d}{dt}(a^{-1}) \propto T_{rel}^{-1} \propto N^{-1}$



[from Merritt+ 2007]

Possible ways to enhance the loss cone repopulation

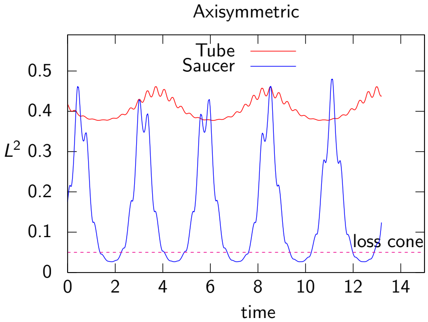
- Brownian motion of the binary (enables interaction with larger number of stars) [Milosavljević&Merritt 2001; Chatterjee+ 2003]
- Non-stationary solution for the loss cone repopulation rate [Milosavljević&Merritt 2003]
- Secondary slingshot (stars may interact with binary several times) [MM03]
- Gas physics – under special circumstances [Lodato+ 2009, Roškar+ 2014]
- Perturbations to the stellar distribution arising from transient events (such as infall of large molecular clouds, additional minor mergers and massive black holes, ...)
- Effects of non-sphericity on the orbits of stars in the nucleus [Berczik+ 2006; Preto+ 2011; Khan+ 2011,2012,2013; Vasiliev+ 2014]

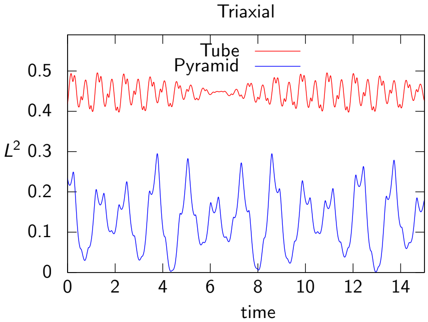
Loss cone in non-spherical stellar systems

Angular momentum L of any star is not conserved, but experiences oscillations due to torques from non-spherical distribution of stars.

Therefore, much larger number of stars can attain low values of L and enter the loss cone at some point in their (collisionless) evolution, regardless of two-body relaxation.

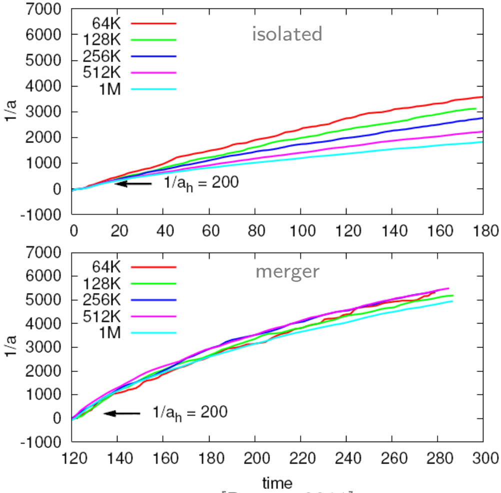
This has led to a conclusion that the loss cone should remain full in axisymmetric and especially triaxial systems.



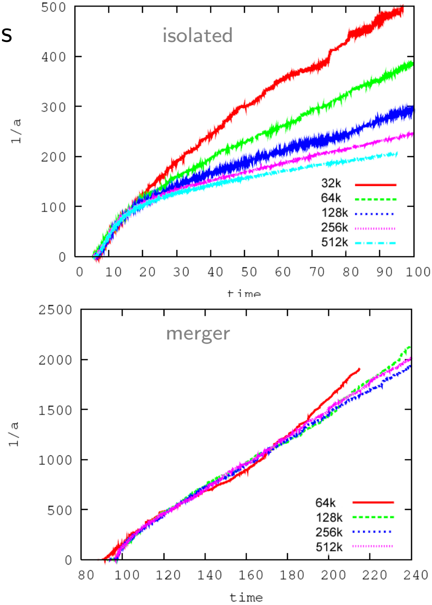


Merger simulations hint for a full loss cone

Hardening rates in merger simulations were found to be N-independent



[Preto+ 2011]



[Khan+ 2011]

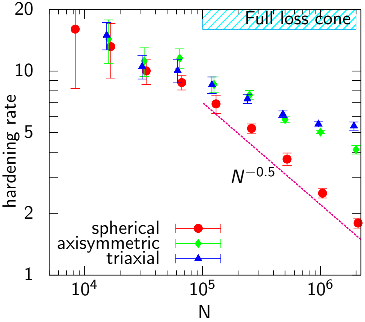
Evolution of isolated systems in different geometries

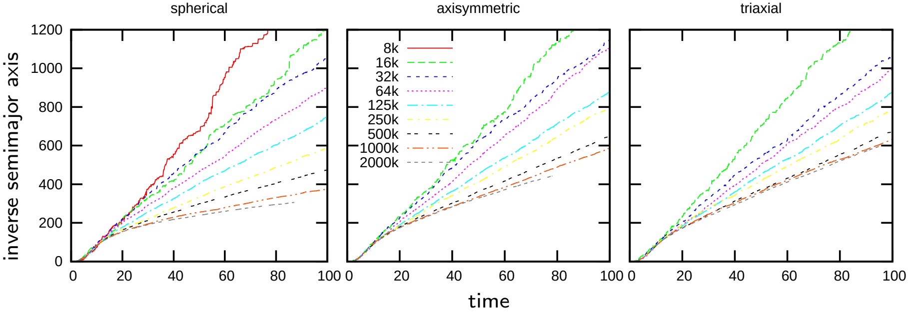
We have performed simulations of binary black hole evolution in three sets of models: spherical, axisymmetric and triaxial.

In all three cases the hardening rate appears to drop with N in the range $10^5 \lesssim N \lesssim 10^6$, but it drops slower in non-spherical cases.

Moreover, this rate is several times lower than the rate that would be expected in the full loss cone regime.

[Vasiliev, Antonini & Merritt 2014]





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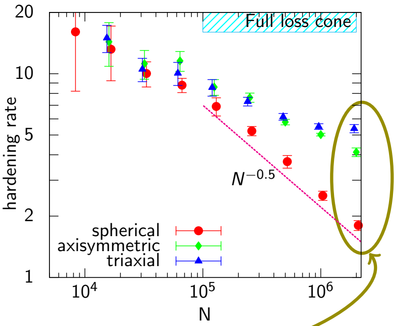
[Vasiliev, Antonini & Merritt 2014]



- Is there a convergence in the limit $N \to \infty$?
- If yes, why the limiting hardening rate seems to be much smaller than the full loss cone value?
- Does it stay constant with time, after all?
- Why the results of merger simulations are different?

Problems with direct N-body simulations

- We model galaxies with $N_\star \sim 10^{10-12}$, but our simulations are feasible only for $N \sim 10^6$;
- therefore, one needs to extrapolate our findings to much higher N;
- but as we have seen, the scaling is non-trivial;
- the contribution of collisional relaxation to loss cone repopulation scales as N^{-1}, but the collisionless processes are independent of N;
- we cannot afford having much larger N even with the best hardware and improved algorithms;
- need a simulation method in which we may adjust the relaxation rate independently of particle number.
- the way to go: combine scattering experiments, evolution of the orbits in a smooth field, and an approximate treatment of relaxation.

[GRAPE cluster in RIT]

Scattering experiments

The interaction between the binary MBH and the field stars may be described as a sum of individual 3-body scattering events.

In each such event, one tracks the changes in energy and angular momentum of the binary [e.g. Sesana+ 2006,2007], or records these changes for the star and adjusts the binary orbit parameters using the conservation laws [Meiron&Laor 2012].

The population of stars that are able to interact with the binary does not stay constant – it needs to be tracked self-consistently:

- stars with low angular momentum are depleted;
- the density cusp around the binary is eroded;
- stars may re-enter the loss cone due to two-body relaxation and non-spherical torques;
- the geometry of non-spherical cusp changes with time.

To address these issues, a model for the evolution of the stellar distribution must be developed.

Self-consistent field method

This method is used for the study of collisionless systems with moderate deviation from spherical shape.

The idea is to expand both density and potential of the system using a suitable (usually orthogonal) set of basis functions which are themselves solutions of Poisson equation (Hernquist&Ostriker 1992):

$$\rho(\mathbf{x}) = \sum_{\mathbf{n}} C_{\mathbf{n}} \rho_{\mathbf{n}}(\mathbf{x}) \; , \quad \Phi(\mathbf{x}) = \sum_{\mathbf{n}} C_{\mathbf{n}} \Phi_{\mathbf{n}}(\mathbf{x}) \; , \quad \nabla^2 \Phi_{\mathbf{n}}(\mathbf{x}) = 4\pi \rho_{\mathbf{n}}(\mathbf{x}) \text{ for } \forall \mathbf{n}.$$

Usually one takes the basis functions to be products of some function in radius and spherical harmonics:

$\Phi_{\mathbf{n}}(\mathbf{r}) = \Phi_{n,l}(r)\, Y_l^m(\theta, \phi)$; $\mathbf{n} \equiv \{n, l, m\}$.

The coefficients of expansion $C_{\mathbf{n}}$ are computed from the positions of all N particles as $C_{\mathbf{n}} = \int d^3\mathbf{x}\, \Phi_{\mathbf{n}}(\mathbf{x})\, \rho(\mathbf{x}) = \sum_{i=1}^{N} \Phi_{\mathbf{n}}(\mathbf{x}_i)\, m_i$.

The simulation workflow is:

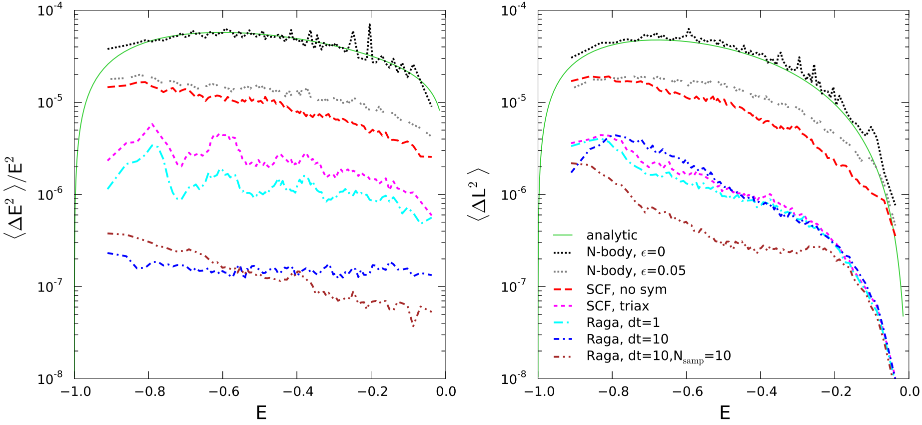
1. Compute the coefficients of potential from particle positions;
2. Move particles according to forces obtained by differentiating the potential, with a timestep $\Delta t \ll T_{\mathrm{dyn}}$.

Self-consistent field method

- This method works rather accurately for systems that have a well-defined center and are well approximated by a moderate number ($\sim$ few dozen) of expansion terms.
- Since particles do not interact with each other explicitly, but their motion is mediated by a smooth potential which represents the mean field, this method is well suited for collisionless simulations.
- However, it is not entirely free of numerical relaxation, since the discreteness noise in the expansion coefficients lead to time-dependent fluctuations in the potential. In fact, the magnitude of numerical relaxation is only a factor of few lower that for other methods with the same N.
- A possible way to reduce fluctuations:
  1. use longer time intervals between updating the potential expansion coefficients (but keep short enough timesteps for integrating the orbits);
  2. during each update interval, store several sampling points for each particle, to increase the effective number of points used in computing coefficients and hence to reduce discreteness noise.

Temporal smoothing in the SCF method

Using a longer interval between potential recomputation and increasing the number of sampling points per particle does help to reduce the artificial relaxation rate by $1 - 2$ orders of magnitude.



Monte Carlo method for collisional stellar systems

Introduced in early 1970s by Spitzer and Hénon as an approximate way of treating two-body relaxation phenomena.
In the formulation of Spitzer&Hart(1971), the equations of motion of each particle in the system are integrated numerically, and after each step a perturbation is applied to the particle's velocity, using first- and second-order diffusion coefficients calculated from the standard two-body relaxation theory:

$$\Delta v_{\parallel} = \langle \Delta v_{\parallel} \rangle \Delta t + \zeta_1 \sqrt{\langle \Delta v_{\parallel}^2 \rangle \Delta t}\,,$$

$$\Delta v_{\perp} = \zeta_2 \sqrt{\langle \Delta v_{\perp}^2 \rangle \Delta t}\,,$$

where ζ_1, ζ_2 are two independent normally distributed random numbers.
The potential of the system is then recomputed using the new positions of all particles. In the original formulation, this has been used in a spherical geometry only, but it is easy to generalize to non-spherical cases, using the SCF approach.

The treatment of two-body relaxation

Local (position-dependent) velocity diffusion coefficients:

$$v\langle\Delta v_{\parallel}\rangle = -\left(1+\frac{m}{m_\star}\right) I_{1/2}\ ,$$
$$\langle\Delta v_{\parallel}^2\rangle = \frac{2}{3}\left(I_0 + I_{3/2}\right),$$
$$\langle\Delta v_{\perp}^2\rangle = \frac{2}{3}\left(2I_0 + 3I_{1/2} - I_{3/2}\right),$$

here m and $m_\star$ are masses of the test and field stars, and

$$I_0 \equiv \Gamma \int_E^0 dE'\, \boxed{f(E'),}$$

distribution function of stars

$$I_{n/2} \equiv \Gamma \int_{\Phi(r)}^{E} dE'\, f(E') \left(\frac{E' - \boxed{\Phi(r)}}{E-\Phi(r)}\right)^{n/2},$$

gravitational potential

$$\Gamma \equiv 16\pi^2 G^2 m_\star \ln\Lambda = 16\pi^2 G^2 M_{\rm tot} \times \boxed{(N_\star^{-1}\ln\Lambda).}$$

scalable amplitude of perturbation

Implementations of the Monte Carlo method

Name	Reference	relaxation treatment	timestep	1:1[1]	BH[2]	remarks
Princeton	Spitzer&Hart(1971), Spitzer&Thuan(1972)	local dif.coefs. in velocity, Maxwellian background $f(r,v)$	$\propto T_{dyn}$	−	−	
Cornell	Marchant&Shapiro (1980)	dif.coef. in E, L, self-consistent background $f(E)$	indiv., T_{dyn}	−	+	particle cloning
Hénon	Hénon(1971)	local pairwise interaction, self-consistent bkgr. $f(r, v_{\parallel}, v_{\perp})$	$\propto T_{rel}$	−	−	
	Stodołkiewicz(1982) Stodołkiewicz(1986)	Hénon's	block, $T_{rel}(r)$	−	−	mass spectrum, disc shocks binaries, stellar evolution
Mocca	Giersz(1998)	same	same	+	−	3-body scattering (analyt.)
	Hypki&Giersz(2013)	same	same	+	−	single/binary stellar evol., few-body scattering (num.)
Cmc	Joshi+(2000)	same	$\propto T_{rel}(r=0)$	+	−	partially parallelized
	Umbreit+(2012), Pattabiraman+(2013)		(shared)	+	+	fewbody interaction, single/binary stellar evol., GPU
Me(ssy)[2]	Freitag&Benz(2002)	same	indiv.$\propto T_{rel}$	−	+	cloning, SPH physical collis.
Raga	this study (Vasiliev 2014)	local dif.coef. in velocity, self-consistent background $f(E)$	indiv.$\propto T_{dyn}$	−	+	arbitrary geometry

[1] One-to-one correspondence between particles and stars in the system

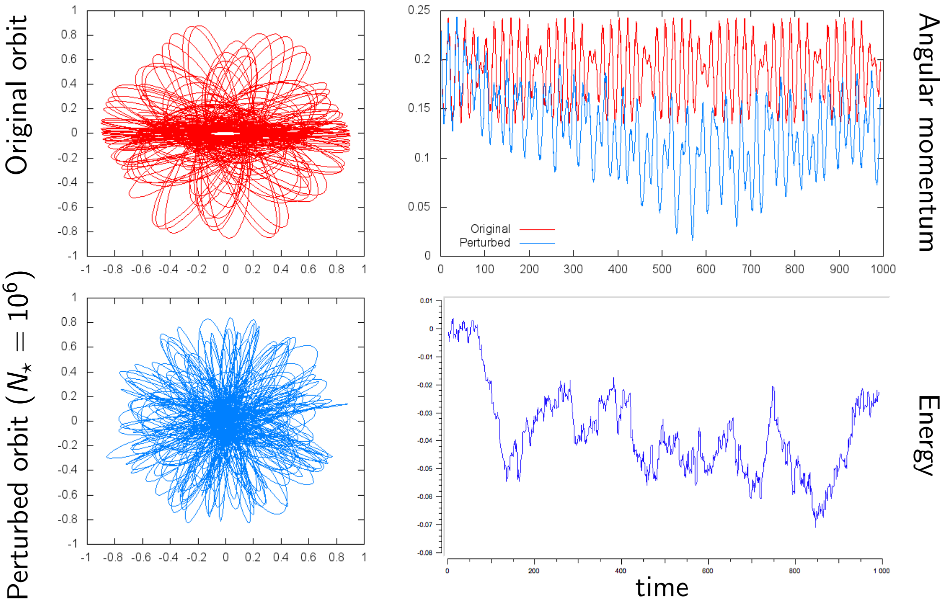
[2] Massive black hole in the center, loss-cone effects

The novel Monte Carlo method for arbitrary geometry

The combination of the temporally-smoothed SCF approach for representing arbitrary non-spherical potential with the Spitzer's formulation of two-body relaxation using local velocity diffusion coefficients leads to a new implementation of the Monte Carlo method suitable for arbitrary geometry.

- **Gravitational potential:**
  particles move in a smooth potential represented by a basis-set expansion.
- **Orbit integration:**
  variable timestep Runge-Kutta; orbits are computed in parallel, independently from each other, during each update interval.
- **Two-body relaxation:**
  apply perturbation to particle velocity using local diffusion coefficients.
- **Potential and distribution function update:**
  update interval $\gg$ dynamical time $\Rightarrow$ temporal smoothing; use many sampling points per particle during each update interval $\Rightarrow$ reduce discreteness noise.

An example of orbit in a triaxial potential

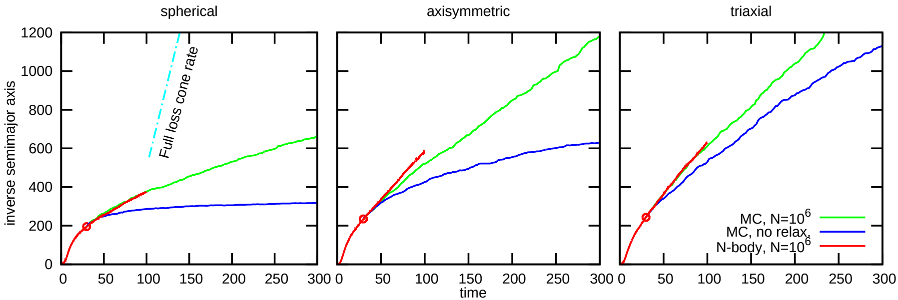


Application to the final-parsec problem

- Follow the merger and initial hardening by a conventional N-body code;
- after the formation of hard binary, switch to Monte Carlo method:
- during each episode, evolve particles in a time-dependent potential of binary MBH moving on a Keplerian orbit with fixed parameters;
- at the end of episode, record the changes of energy and angular momentum of each particle during each close encounter with the binary, sum them up and adjust the orbit of the binary using conservation laws [e.g. Sesana+ 2007, Meiron&Laor 2012];
- this automatically accounts for depletion and repopulation of the loss cone, secondary slingshot, and change of shape of the gravitational potential; does not account for brownian motion;
- may also include two-body relaxation in addition to non-spherical torques ⇒ naturally interpolate between $N = 10^6$ and $N = \infty$.

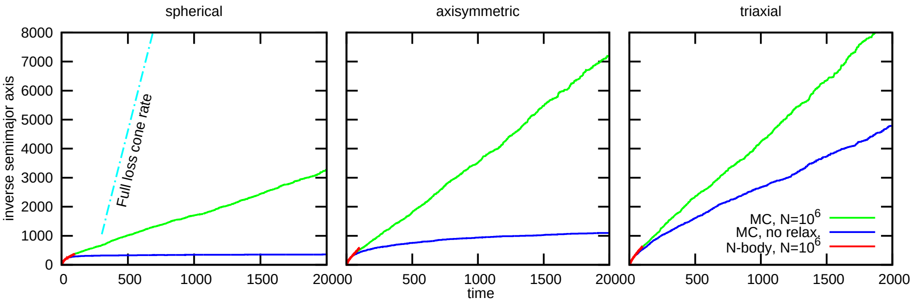
Preliminary results of Monte Carlo simulations

- Monte Carlo simulations are in qualitative agreement with direct N-body simulations, although somewhat underestimate hardening rate.
- Hardening rate does not stay constant, but decreases with time; it is never even close to the full loss cone rate.



Preliminary results of Monte Carlo simulations

- Monte Carlo simulations are in qualitative agreement with direct N-body simulations, although somewhat underestimate hardening rate.
- Hardening rate does not stay constant, but decreases with time; it is never even close to the full loss cone rate.
- It's not clear if the "final parsec" gap can be overcome in the purely collisionless axisymmetric case [in contrast with Khan+ 2013].
- In the triaxial case there is little "benefit" from relaxation.



Summary

- The final parsec problem in the binary MBH evolution is connected to the efficiency of repopulation of the loss cone.
- This repopulation occurs faster in non-spherical geometry.
- In simulations of isolated systems, the loss cone never stays "full", even in triaxial geometry.
- It is difficult to disentangle collisional and collisionless effects (suppress 2-body relaxation) in conventional N-body simulations.
- A novel Monte Carlo method with arbitrary geometry and adjustable relaxation rate is proposed.
- Preliminary results suggest that the final parsec problem may be overcome only in triaxial systems (axisymmetry is not enough).
- Remaining problem: apparently different situation (higher hardening rate) in merger simulations.

Thank you!